Grupo de Pesquisa em
**Gestão e Planejamento Econômico-Financeiro**
**Universidade Federal do Rio de Janeiro – UFRJ**

# A Simple Structural Macro-Econometric Model Applied to Financial Investment Decisions: Taking Keynes to the 1980s

Manuel A. R. da Fonseca

**Textos para Discussão**

**No. 3 – fev. 2017.**

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**Título**

# A Simple Structural Macro-Econometric Model Applied to Financial Investment Decisions: Taking Keynes to the 1980s

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**Resumo:** O objetivo central deste texto é desenvolver o sistema analítico de Keynes em um modelo macro-econométrico dinâmico, e este modelo é posteriormente aplicado à análise dos principais indicadores macroeconômicos dos EUA no período entre 1954 e 1984. Além disso, a solução dinâmica gerada pelo modelo macro-econométrico é comparada com dados para carteiras bem conhecidas com o objetivo de avaliar se o modelo poderia ter produzido informação relevante para decisões de investimento nestas carteiras. Mais especificamente, a questão colocada é: Alguém poderia ter ganhado dinheiro através deste modelo simples? O argumento apresentado no texto indica que a resposta apropriada é *sim*, *talvez*. No processo de desenvolver e aplicar o modelo macro, o sistema analítico de Keynes é revisto, assim como métodos econométricos usados para estimação de parâmetros de equações estruturais, e métodos matemáticos para a solução de sistemas de equações não lineares.

**Palavras-chave**: Modelos macro-econométricos, Métodos para estimação e solução de sistemas de equações, Projeções macroeconômicas; Decisões de investimento financeiro.

**Abstract:** The main goal of this paper is to develop the standard Keynes' analytical system into a dynamic macro-econometric model, which is then applied to the analysis of the main macroeconomic indicators for the US in the period from 1954 to 1984. Further, the dynamic solution generated from the macro-econometric model is contrasted with data for some well-known financial portfolios with the purpose of assessing if the model could have produced relevant information concerning investment decisions on these portfolios. More directly, the question proposed is: One could have made money using such a simple model? The argument put forward in the paper indicates that the appropriate answer is *yes*, *maybe*. In the process of developing and applying the macro model, the Keynesian analytical system is reviewed, as well as econometric methods used for parameter estimation of structural models, and mathematical methods for the solution of nonlinear systems of equations

**Key-words**: Macro-econometric modeling; Methods for estimation and solution of systems of equations; Macroeconomic forecasts; Financial investment decisions.

# A Simple Structural Macro-Econometric Model Applied to Financial Investment Decisions: Taking Keynes to the 1980s

Manuel A. R. da Fonseca[*]

"I do not know of a complicated model in any area of science that performs well in explanation and prediction [...]. So far, I have not heard about a single one. Certainly the large scale econometric models and complicated VARs […] have not been very successful in explanation and prediction. Thus it appears useful to start with a well understood, *sophisticatedly simple* model and check its performance empirically in explanation and prediction."

Arnold Zellner[**]

ABSTRACT

The main goal of this paper is to develop the standard Keynes' analytical system into a dynamic macro-econometric model, which is then applied to the analysis of the main macroeconomic indicators for the US in the period from 1954 to 1984. Further, the dynamic solution generated from the macro-econometric model is contrasted with data for some well-known financial portfolios with the purpose of assessing if the model could have produced relevant information concerning investment decisions on these portfolios. More directly, the question proposed is: One could have made money using such a simple model? The argument put forward in the paper indicates that the appropriate answer is *yes*, *maybe*. In the process of developing and applying the macro model, the Keynesian analytical system is reviewed (Section 1), as well as econometric methods used for parameter estimation of structural models (Section 2), and mathematical methods for the solution of nonlinear systems of equations (Section 3). The part dealing more specifically with macroeconomic and financial data appears in Section 4.

KEY WORDS: Macro-econometric modeling; Methods for estimation and solution of systems of equations; Macroeconomic forecasts; Financial investment decisions.

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[**] Interview for the *International Journal of Forecasting*, quoted in B. Saffran, "Recommendations for Further Reading", *Journal of Economic Perspectives*, v. 13, no. 2, 1999, pp. 231-37 (originally without italics).

## Introduction

A clear observable fact is that mainstream macroeconomics – although the meaning of this expression is by no means certain – has evolved in the last few decades in the direction of bigger and more complex models devised to understand and analyze the behavior of the overall economy, and to make relevant conclusions about economic policy and future developments. As important examples of this statement, one can mention the predominance of large scale DSCE (Dynamic Stochastic General Equilibrium) models, as well as of VAR (Vector Auto-Regression) models. A problem that many economists and professionals perceive in these models is that they are not very successful in predicting future developments in macroeconomic aggregates, so that their practical use has been increasingly reduced – especially in financial applications, where decisions have outright implications in terms of making or losing money.

Considering this predominant trend in Macroeconomics, this paper's main goal implies a move – quite radical, in fact – in the opposite direction. The objective is to use a structural macro-econometric model with barely a few simple equations, and evaluate its performance in reproducing major macroeconomic trends.

One of the pioneers in the field of structural macro-modeling was Lawrence R. Klein, who, in the 1940s, developed a six-equation dynamic macro-model, which became known as Klein Model I.[1] From the 1960s to 1980s, many well-known Econometrics textbooks contained thorough material on structural macro-dynamic models. One good example is Pindyck and Rubinfeld (1981). More recently, Greene (1997) includes a fairly detailed analysis of Klein Model I. A brief and interesting text that applies the developments in this field to macroeconomic planning is Heesterman (1970).

The model used here is developed from the standard Keynes' analytical system, and is then applied to the analysis of the main macroeconomic indicators for the US in the period from 1954 to 1984.[2] Further, the dynamic solution generated from the macro-econometric model is contrasted with data for some common financial portfolios with the purpose of assessing if the model could have produced relevant information concerning investment decisions on these portfolios.

In the first Section, the Keynesian analytical system is reviewed and the econometric equations of the macro dynamic model are introduced. In Section 2, econometric methods used for parameter estimation of structural models are briefly reviewed, and the estimates for the macro-econometric model are provided. Mathematical methods for the solution of nonlinear systems of equations are summed up in Section 3, and the nature of the solutions that were obtained is discussed. The part dealing more specifically with macroeconomic and financial data appears in Section 4.

[1] The details on the data base, parameter estimation and solution of this model appear in Klein (1950). For more recent developments, one reference is Klein and Young (1982). In 1980, Klein was awarded the Nobel Prize for his contributions in the development and application of econometric models.
[2] It appeared previously in Fonseca (2005 and 2006).

## 1. A simple model that transformed our understanding of market economies – and an econometric counterpart

From the involved and non-mathematical exposition in Keynes' General Theory, published in 1936, one can deduce a fairly simple and straightforward model with equations representing three overall markets: for goods, money and labor. The model's equations are specified below:[3]

I. Equilibrium output and aggregate production function

$$Y = C + I + G + (EX - IM) \tag{1}$$

$$Y = f(L) \tag{2}$$

II. Effective demand

$$C = C(Y - T) \tag{3}$$

$$I = I(r) \tag{4}$$

III. Nominal values

$$M = M(Y, r)P \tag{5}$$

$$w = \left[\frac{d}{dL} f\right] P \tag{6}$$

This system of equations is a staple tool in the Macroeconomics field and scarcely needs any further explanation. In any case, a description of the endogenous and exogenous variables follows:

Endogenous variables:

- $Y$: Aggregate income and product, in real values;
- $L$: Total employment (number of persons);
- $C$: Private consumption, in real values;
- $I$: Aggregate investment, in real values;
- $r$: Interest rate, in real terms;
- $P$: Average price level.

Exogenous variables:

- $G$: Government spending, in real values;
- $X–M$: Trade balance in national currency, real values;
- $T$: Taxes, real values;
- $M$: Money stock, current values;
- $w$: Average wage rate, current values.

To a large extent, the contents of any standard Macroeconomics textbook are related to the concepts and analyses arising from these six equations. In especial, the combination of equations (1), (3), (4), and (5) produces the standard IS-LM analysis of output and interest rate equilibrium. On the other hand, Equations (2) and (6) are usually dealt with

[3] Although virtually any macroeconomic textbook contains the topics in this Section, the reference used here is Allen (1968), chap. 7.

independently of the rest of the system and, therefore, the price level and the labor force employed are treated in textbook analyses as exogenous elements.

In order to explore some of the economic rationality in this model, we can consider, for example, what would happen in a country where money stock (*M*) is allowed to increase. An initial effect would be a reduction of *r*, determined by eq. (5).[4] Then, given eq. (4), aggregate investment (*I*) would rise, leading to an increment in *Y* (eq. 1). This would cause an increase in *C* and further increments in *Y* through the multiplier effect. On the other hand, eq. (2) shows that a larger output depends upon a greater use of labor. However, from the standpoint of firms' profit maximization, a necessary condition for an increase in employment is that the cost of labor should be reduced, in order to compensate for the lower marginal productivity of labor (eq. 6, given that $f'' < 0$). This means that larger output and employment depend upon an increment of *P*, so that the real wage rate is reduced (workers behave as if they suffered from monetary illusion, worrying only about the nominal wage rate *w*, at least in the short run). Thus the ultimate effects of an increase of *M* are increments in *Y*, *C*, *I*, *L*, and *P*, accompanied by a decrease of *r*.

Alternatively we can explore the consequences, derived from this six-equation system, of an increment in *G*. An initial effect, given eq. (1), would be a rise of *Y*. Moreover, if *M* is held fixed, eq. (5) indicates that *r* should increase, leading to a reduction of *I* (eq. 4). These opposing forces compensate each other, leaving *Y* relatively unchanged – the so-called *crowding out* effect of higher government spending. However, if *M* is allowed to increase in order to accommodate the demand pressures set off by an enlarged government spending, then the final results are very much like those derived in the previous paragraph.

In the sequence, a structural macro-econometric model based on Keynes' six-equation system is developed – a model that includes variables in both real and nominal values. To start, eq. (2) can be used in one of the other equations so that it can be eliminated – which makes sense given that the empirical development of an aggregate production function is an endeavor not commonly pursued. Introducing a relation for total employment, $L = f^{-1}(Y)$, and substituting in eq. (6), we get:

$$w = g(Y)\,P \qquad (7)$$

Further, for the Keynesian system, one can introduce econometric equations that can be estimated using standard methods – that is, equations that are linear in the *parameters*. These econometric equations, however, are nonlinear in the endogenous *variables* – which means that one must use mathematical methods designed to solve nonlinear systems of equations.

IV. Econometric equations

$$Y = C + I + G + (EX - IM) \qquad (1)$$

$$C = \alpha_0 + \alpha_1 (Y - T) + \varepsilon_C \qquad (3a)$$

$$I = \beta_0 + \beta_1 r + \beta_2 \Delta Y + \beta_3 Y_{-1} + \varepsilon_I \qquad (4a)$$

[4] The derivatives of the functions are not included, but this is straightforward information.

$$\frac{M}{P} = \gamma_0 + \gamma_1 r + \gamma_2 Y + \varepsilon_M \quad \rightarrow \quad r = \left(\frac{M}{P} - \gamma_0 - \gamma_2 Y - \varepsilon_M\right)\frac{1}{\gamma_1} \qquad (5a)$$

$$\ln\left(\frac{w}{P}\right) = \zeta_0 + \zeta_1 \ln Y + \varepsilon_w \qquad (7a)$$

Eq. (4*a*) includes an endogenous variable with a one period delay and, consequently, this set of equations is a dynamic system – technically, a system of difference equations.[5] Therefore, this is a structural nonlinear dynamic macro-econometric model – a long and pompous name for a system with only five equations. But the fundamental question remains: Is it possible that such a simple system can be of value in analyses of the real world? More importantly: Can it contribute to the process of designing successful financial investment strategies? Before we can try to answer, the equations must be estimated and solved. These developments are pursued in the next two sections.

## 2. Data set and estimation of the parameters – econometric methods for structural equations

Usually one of the most difficult and challenging tasks in the development of a macro-econometric model is to assemble the data base for its variables. However, in the case of the model developed in this paper – quite frankly – this was one of the easiest parts. The series used were drawn from data available on the covers of a pair of traditional textbooks – appropriately, one of the texts is on (Macro) Economics and the other on Econometrics.[6] These series appear in Appendix 1.

The estimation method used here is the Two-Stage Least Squares (2SLS) estimator, which is a particular case of the more general method known as Instrumental Variables (IV) estimator – which, in turn, can be considered an especial case of the Method of Moments applied to regression analysis.[7] It can be shown that estimators obtained from these methods have desirable asymptotical properties – further, under certain conditions, they are asymptotically efficient. The basic formula for the 2SLS estimator, applied to the *i*–th equation ($\bar{\delta}_i$), is:

$$\bar{\delta}_i = (\bar{Z}_i^T \bar{Z}_i)^{-1} \bar{Z}_i^T y_i \qquad (8)$$

In eq. (8), the data for the corresponding endogenous variable are included in $y_i$. Matrix $\bar{Z}_i$ is defined as:

$$\bar{Z}_i = [\hat{Y}_i \,|\, X_i] \qquad (9)$$

In this matrix, $X_i$ contains the data for the exogenous (and lagged endogenous) variables included in the *i*–th equation, and the $\hat{Y}_i$ part contains the *instruments* – forecasts for the endogenous variables in the *i*–th equation, which were are generated using the model in

[5] Eq. (4*a*), which shapes the dynamics of the model, plays a unique role in this system – it is actually an example of the “art” of specifying econometric equations. If the model’s dynamics works appropriately, it should reproduce with some success the time path of the real macroeconomic variables.
[6] The references are: W. J. Baumol and A. S. Blinder, *Economics – Principles and Policy*, 3.ª ed., and Greene (1997). In the first reference’s cover, there are data for *Y*, *C*, *I*, *G*, (*EX–IM*), nominal *r*, *w* and *P*. In the cover of the second, there are data for (*Y–T*) and *M* / *P*.
[7] Campbell, Lo and MacKinlay (1997).

its *reduced form*.[8] All the estimation procedures for the model's equations were developed in a spreadsheet file.

The parameters were estimated using data series for the 1953-79 period (Appendix 1) – the last five data available in each series were not used. The reason is that this part of the data was reserved so that it could be used to evaluate the model as a prediction tool – that is, beyond the period used in the estimation process. The estimates appear in Table 1.

**Table 1. Parameter estimates – 2SLS estimator**
Data series: 1953-79.

| | | | | |
|---|---|---|---|---|
| **Dependent variable: *C*** | | | | |
| Regressors | 1 | $Y–T$ | | |
| Coefficient | 8.052 | 0.8947 | | |
| Standard error | 5.297 | 0.0076 | | |
| **Dependent variable: *I*** | | | | |
| Regressors | 1 | $r$ | $\Delta Y$ | $Y_{-1}$ |
| Coefficient | –6.437 | –3.3935 | 0.5975 | 0.1503 |
| Standard error | 12.591 | 4.2453 | 0.1182 | 0.0095 |
| **Dependent variable: *r*** | | | | |
| Regressors | 1 | $M/P$ | $Y$ | |
| Coefficient | 1.1619 | –0.0173 | 0.00423 | |
| Standard error | 0.9955 | 0.00978 | 0.00277 | |
| **Dependent variable: ln(*w/P*)** | | | | |
| Regressors | 1 | $\ln Y$ | | |
| Coefficient | 1.8317 | 0.3870 | | |
| Standard error | 0.1551 | 0.0226 | | |

## 3. Solving a system of nonlinear equations – each simulation period contains a problem to be solved

Solution methods of nonlinear systems of equations are based upon – with a decreasing level of generality – fixed-point theorems, the method of successive approximations, and the so-called Newton (or Newton-Raphson) method.[9] Fixed-point theorems establish that, under certain conditions, the solution of the equation $x = f(x)$ exists. One important aspect of this analysis is that any equation can be put in the form of the fixed-point equation. For example, if the original equation is $g(x) = 0$, then one corresponding $x = f(x)$ equation is:

$$x = x + g(x) \qquad (10)$$

[8] Maddala (1987), chap. 11.
[9] Franklin (1980), chap. 3, and Strang (1986), chap. 5.

The method of successive approximations is, at the same time, one way to prove the theorem and a method to find the solution. In this iterative scheme, one computes the successive approximations:[10]

$$x_{n+1} = f(x_n); \qquad n = 0, 1, 2, \ldots \qquad (11)$$

The method associated to Newton is an important example of the iterations in (11). Representing the $i$–th equation in a nonlinear system by $g_i(\boldsymbol{x}) = 0$, where the symbol $\boldsymbol{x}$ represents the set (vector) of endogenous variables, the method provides the solution in each stage – or approximation – ($\boldsymbol{x}^{k+1}$) from the previous one ($\boldsymbol{x}^k$) through the following linear system:

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - (J^k)^{-1}\,\boldsymbol{g}^k \qquad (12)$$

In eq. (12), $J^k$ represents the Jacobian matrix (of first derivatives) obtained in stage $k$, and $\boldsymbol{g}^k$ is a vector with the results calculated at the same stage for the functions $g_i$. Usually, the Newton method converges quickly for the correct solution. One difficulty with this method is that the Jacobian matrix must be calculated at each step and, naturally, in a more convenient version of the method – albeit one in which convergence to the solution is not so quick, neither so much guaranteed – this matrix is fixed. In the so-called *modified* Newton method, only the Jacobian matrix for the first stage (0) is used:

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - (J^0)^{-1}\,\boldsymbol{g}^k \qquad (13)$$

The modified Newton method is quite easily translated into computer code – in the case of the model solved here, a spreadsheet was used to provide the solutions. This arrangement is very convenient, since the construction of data series for the variables, the estimation of the equations' parameters, and the model's dynamic solution can all be performed in different spreadsheets of the same file.

The first period for which the model was solved was 1954, using data for the previous year – which also provided a starting solution for the successive approximations method. After that, the solution obtained in one period was used as a starting point for the following year. In each period, the modified Newton method was utilized and, as a rule, only a few iterations were necessary to achieve convergence. Therefore, in terms of data for the endogenous variables, only information for 1953 was, in fact, used. In other words, the values for the endogenous variables obtained from the model reflect only the dynamic properties of the nonlinear system, and the trend of the exogenous variables – which are quite limited in number (only five).

The solutions obtained from the five-equation macro-econometric model, together with the historical series, are displayed in Figure 1.

[10] To prove the fixed-point theorem, it is sufficient to establish the convergence of the sequence $x_n$.

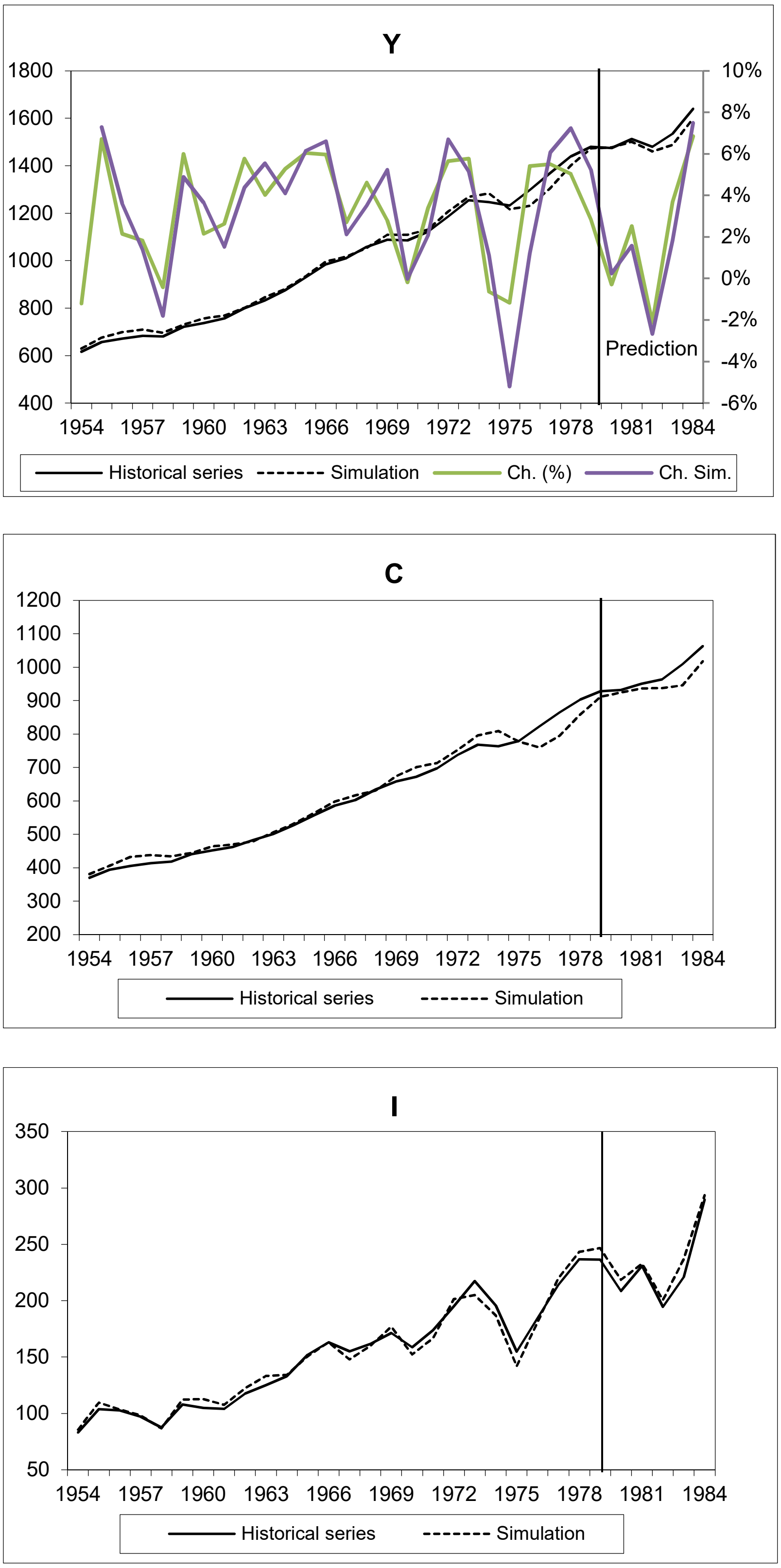


**Figure 1. Model's solutions and historical series.**

Notes: 1. For the units used in each graph, see Appendix 1.

2. The prediction period contains "out of sample" data – *i. e.*, data that were not used in parameter estimation.

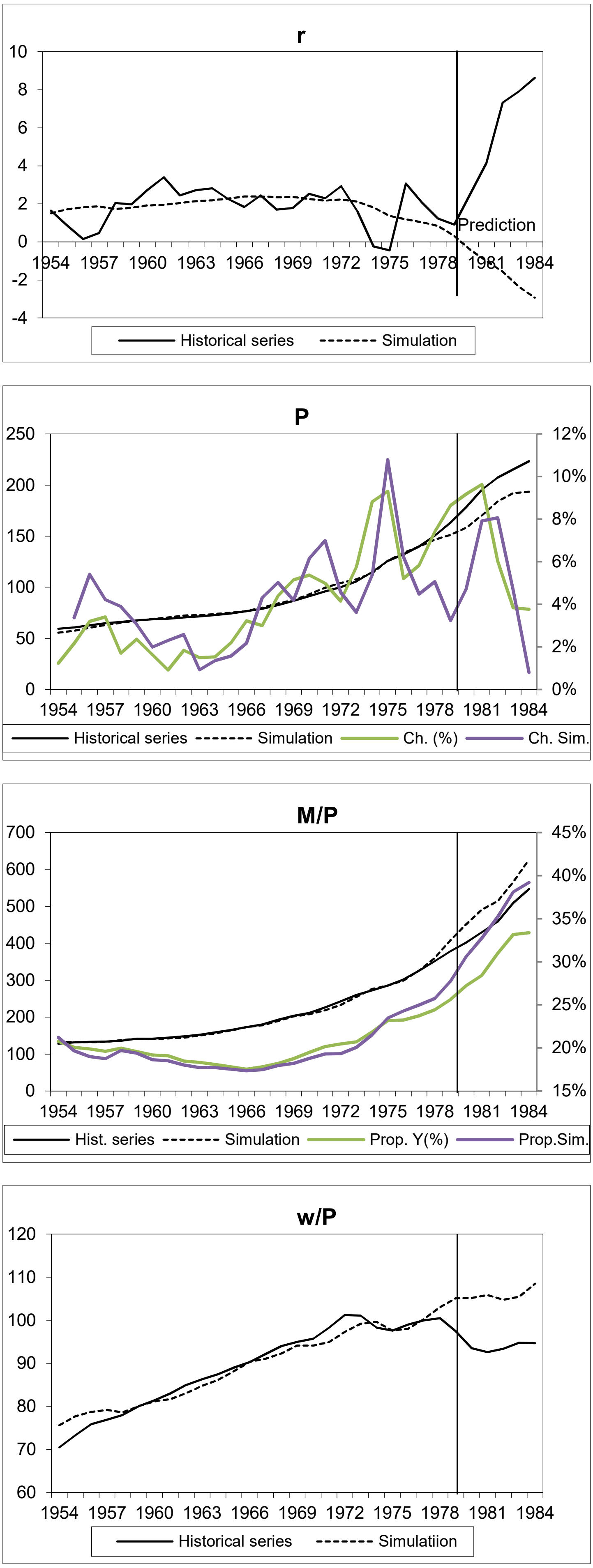


**Figure 1 (cont.)**

### 3.1. A few comments on the dynamic solutions

In Figure 1, the most significant graph is probably the one for aggregate investment (*I*) – the true dynamic variable in the model. Based on this graph, one can conclude that the model captured fairly well the dynamic pattern of the real economy.[11] On the other hand, one should not over emphasize this aspect of the model's solutions – that they were quite close to historical values –, given the especial nature of the exogenous variables. Although these variables are limited in number, each one represents fundamental and complex parts of the macroeconomic system.

For example, the solutions for the endogenous variables depend on the path of the nominal wage rate (*w*), which should be forecasted independently of the macro-model. Also, to predict the future path of trade balance (*EX–IM*), one would likely need a specific model oriented for the external sector. In any case, having said this, one favorable aspect of the macro-econometric model is that, if it were in fact used in financial investment decisions, the investor would only need to predict the exogenous variables a few years ahead. Further, different scenarios could be devised, and the investment decisions could be revised as one of these scenarios revealed itself more likely to prevail in the future.

Moreover the most valuable feature of a structural macro-econometric model is that it reveals, among hundreds of different types of information available to investors, what we should be concentrating on. More explicitly, if the model reproduces reasonably well the pattern of real macroeconomic variables, and given that the model's solutions derive from the exogenous variables and the model's dynamic properties, then it is a logical conclusion that these exogenous variables are the really important ones for financial investors – and there are only five of them to keep track of.

In other words, one of the main accomplishments of this macro-econometric model is that it reveals that financial investors should pay especial attention to the trends of average nominal wage, trade balance, government spending, taxes, and money supply. Further, the model produces *sophisticatedly simple* forecasts from the projected path of these variables – forecasts that can be of value in designing financial investment strategies.

## 4. Contrasting macroeconomic data with portfolios' returns – assessing the model's potential use in financial investment decisions

In this part, macroeconomic data and forecasts are contrasted with annual returns of general portfolios – the goal is to access if the model could have produced useful information for investors. Financial data appear on Appendix 2. Dealing with stocks first, let us concentrate on data for firms with large capitalization values, which show a more regular pattern. In the period from 1950 to 1990, the five highest returns – for the full year – correspond to 54 (52.55%), 58 (43.78%), 75 (37.26%), 80 (32.48%), and 85 (32.00%). On the opposite side, the worse full-year results occurred in 74 (–26.4%), 73 (–14.7%), 57 (–11.14%), 66 (–10.25%), and 62 (–8.79%). As can be perceived, the upper side results were much higher than the negative ones – a trend that helped

[11] Putting it differently, the econometric "artist" was successful.

investor W. Buffet, for example, to impressively increase the value of his holding company (Berkshire Hathaway).

Examining the downside first, as we can see from the first graph in Figure 1 (*Y*), in the 1973-74 period, there was a significant worsening of GNP, which moved from a growth rate close to 6% to a slightly negative rate. The scenario was about the same in 1975, but there was a strong recovery in 76, with a growth rate above 5%. Thus, the strong gain of the S&P index in 1975 (37.26%) reveal that the stock market correctly anticipated the GNP recovery in the following year. Similarly, in the 1955-58 span, there also was a major GNP growth downturn, with the rate moving sharply lower from close to 7% (in 55) to 2%, and then below 2%, until becoming slightly negative in 58. In 1959, this trend reversed, and the growth of GNP reached 6%. Given the strong number for the S&P return in 1958 (43.78%), we can conclude that, once again, the market correctly anticipated the recovery that would materialize the following year.

The trends examined in the previous paragraph, in which movements in the S&P index accompanied – actually, anticipated – the changes of economic growth, were also present in 1962 and 1966. On the other hand, in the strong performance of the S&P index in 1980, the stock market was short-sided, given the disappointing growth in the following year. Considering the model's solutions, the most striking fact is that all these changes in GNP growth were also present in these solutions.

The series with rates of change of GNP (*Y*) and annual returns of the S&P 500 are combined in Figure 2. As can be perceived, as a rule, the stock market anticipated the behavior of GNP growth – usually, changes in the rates of return for the S&P 500 are one year ahead the corresponding changes in GNP growth rates. Therefore, Figure 2 indicates that there is a particularly important form of market efficiency – the stock market anticipates in at least one year the behavior of main macroeconomic aggregates.

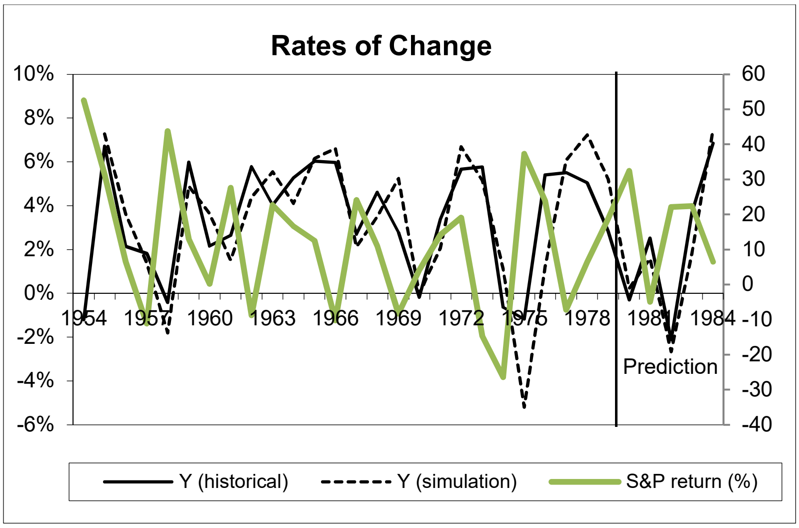


**Figure 2. Model's solutions and historical series for GNP, and S&P 500 annual returns.**

On important conclusion of the latter analysis is that, if an investor intends to make money by trying to beat the stock market, he or she needs to accurately predict macroeconomic trends more than one year in advance – therefore, the investor will need all the help he or she can get, perhaps including the use of an adequate structural macro-econometric model

On the other hand, considering the portfolios of long run and intermediate Treasury bonds – the annual returns appear in Appendix 2 –, it is quite clear that the macro-econometric model would be inefective in setting up strategies for investment decisions in debt securities. The reason is the very disappointing record of real interest rates forecast, as can be seen from Figure 1 (graph for *r*).

## 5. Concluding remarks

The analyses in the previous Sections indicate that structural macro-econometric models – even very simple ones, as the model examined here – can be valuable tools in the process of forecasting main macroeconomic variables and, in this way, they contain relevant information for financial market investors – especially in relation to the stock market. On the other hand, it is important to emphasize that, as one would expect, there is no "magic bullet" here. Even though sophisticatedly simple macro-dynamic models can combine information in a systematic way, and generate solutions that would not normally be available otherwise, these solutions are derived from exogenous variables that depend on involved macroeconomic relations.

One drawback of structural macro-econometric models is that, before they can be applied to problems in the economic field and in Finance, a number of previous tasks must be successfully completed, in particular: development of relevant macro-econometric equations, building of data bases for the model's variables, estimation of the parameters using adequate econometric methods, and the application of mathematical methods for the solution of nonlinear systems of equations – in this case, there is also the need to write adequate computer code. In relation to these points, one favorable aspect is that sophisticated software tools are currently available to deal with all these tasks.

In the period that was examined here, the macro-econometric model based on Keynes was very successful in reproducing the endogenous variables, with the significant exception of the real exchange rate (*r*). Moreover, it was clearly perceived that, in this period, there was a close relation between the behavior of the rate of change of GNP (*Y*) and annual returns of the S&P 500 index – actually, as a rule, the stock market anticipated the trend in GNP growth by at least one year. The main conclusion is that there is convincing evidence that a simple structural dynamic macro-econometric model, as the one developed here, can be of help in designing investment strategies for the stock market.

### 5.1. Some afterthoughts

At the end of 2016, a political event that has caused heated debates and conflicting perspectives was the election of an unorthodox candidate as President of the US. Shortly after the election, there was a significant development in the stock market that apparently resulted from it – a major bullish movement that lasted for several weeks. According to some sources, roughly $1.5 trillion dollar of wealth was created in the stock market from early November until the beginning of January – W. Buffet alone had a wealth gain of $6.7 billion.[12]

Such a development has puzzled many experts and people in general, and yet it is in agreement with the conclusions based on the macro-econometric model examined here. More specifically, notwithstanding the polemic content of some proposals and the regrettable discourse, the emphasis of the new American administration on improving the country's trade balance, on greater government spending in infrastructure, and on tax reduction should lead to higher economic growth – and the stock market, as it is usually the case, correctly anticipated this development.

[12] "10 people profit $8.9 billion from Trump rally". *USA TODAY*, Jan. 11, 2017.

**Appendix 1:**
**Data for major macroeconomic variables: USA, 1953-84**[1]

| | *Y* | *C* | *I* | *G* | *(EX-IM)* | *(Y-T)* | $r_{nom}$ | $P^3$ | $M/P^4$ | $w^5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1953 | 623.6 | 363.4 | 85.3 | 170.1 | 4.8 | 399.1 | 1.62 | 58.82 | 126 | 39.93 |
| 1954 | 616.1 | 370 | 83.1 | 156 | 6.9 | 403.6 | 1.64 | 59.55 | 128 | 41.48 |
| 1955 | 657.5 | 394.1 | 103.8 | 152.3 | 7.3 | 427 | 0.87 | 60.84 | 132 | 44.07 |
| 1956 | 671.6 | 405.4 | 102.6 | 153.5 | 10.1 | 446.5 | 0.15 | 62.79 | 133.5 | 47.09 |
| 1957 | 683.8 | 413.8 | 97 | 161.2 | 11.8 | 455.2 | 0.47 | 64.93 | 134.1 | 49.34 |
| 1958 | 680.9 | 418 | 87.5 | 169.8 | 5.6 | 461 | 2.05 | 66.04 | 136 | 50.9 |
| 1959 | 721.7 | 440.4 | 108 | 170.6 | 2.7 | 479.3 | 1.97 | 67.6 | 141.4 | 53.44 |
| 1960 | 737.2 | 452 | 104.7 | 172.8 | 7.7 | 489.6 | 2.74 | 68.7 | 141.4 | 55.26 |
| 1961 | 756.6 | 461.4 | 103.9 | 182.9 | 8.5 | 503.9 | 3.4 | 69.33 | 144.5 | 56.86 |
| 1962 | 800.3 | 482 | 117.6 | 193.2 | 7.5 | 524.8 | 2.44 | 70.61 | 148 | 59.31 |
| 1963 | 832.5 | 500.5 | 125.1 | 197.6 | 9.4 | 542.7 | 2.72 | 71.67 | 152.6 | 61.12 |
| 1964 | 876.4 | 528 | 133 | 202.6 | 12.8 | 580.5 | 2.82 | 72.77 | 158.6 | 62.92 |
| 1965 | 929.3 | 557.5 | 151.9 | 209.8 | 10.1 | 616.3 | 2.26 | 74.36 | 165.5 | 65.4 |
| 1966 | 984.8 | 585.7 | 163 | 229.7 | 6.5 | 647 | 1.84 | 76.76 | 172.8 | 68.49 |
| 1967 | 1011.4 | 602.7 | 154.9 | 248.5 | 5.4 | 673.1 | 2.44 | 79.06 | 180 | 72.03 |
| 1968 | 1058.1 | 634.4 | 161.6 | 260.2 | 1.9 | 701.4 | 1.7 | 82.54 | 192.7 | 76.67 |
| 1969 | 1087.6 | 657.9 | 171.4 | 257.4 | 0.9 | 722.7 | 1.79 | 86.79 | 203.8 | 81.47 |
| 1970 | 1085.6 | 672.1 | 158.5 | 251.1 | 3.9 | 751.7 | 2.53 | 91.45 | 211.6 | 86.48 |
| 1971 | 1122.4 | 696.8 | 173.9 | 250.1 | 1.6 | 779.1 | 2.29 | 96.01 | 226.2 | 93.26 |
| 1972 | 1185.9 | 737.1 | 195 | 253.1 | 0.7 | 810.3 | 2.93 | 100 | 242.6 | 100 |
| 1973 | 1254.3 | 767.9 | 217.5 | 253.3 | 15.5 | 865.2 | 1.6 | 105.75 | 259.7 | 105.65 |
| 1974 | 1246.3 | 762.8 | 195.5 | 260.3 | 27.8 | 857.7 | - | 115.08 | 272.6 | 111.78 |
| 1975 | 1231.6 | 779.4 | 154.8 | 265.2 | 32.2 | 874.8 | - | 125.79 | 285.4 | 121.32 |
| 1976 | 1298.2 | 823.1 | 184.5 | 265.2 | 25.4 | 906.9 | 3.06 | 132.34 | 301.9 | 129.46 |
| 1977 | 1369.7 | 864.3 | 214.2 | 269.2 | 22 | 943.3 | 2.07 | 140.05 | 325.2 | 138.39 |
| 1978 | 1438.6 | 903.2 | 236.7 | 274.6 | 24 | 988.6 | 1.23 | 150.42 | 351.7 | 149.38 |
| 1979 | 1479.4 | 927.6 | 236.3 | 278.3 | 37.2 | 1015.5 | 0.91 | 163.42 | 379 | 157.28 |
| 1980 | 1475 | 931.8 | 208.5 | 284.3 | 50.3 | 1021.7 | 2.53 | 178.42 | 401.5 | 164.84 |
| 1981 | 1512.2 | 950.5 | 230.8 | 287 | 43.8 | 1049.7 | 4.14 | 195.6 | 430.1 | 178.98 |
| 1982 | 1480 | 963.3 | 194.4 | 292.7 | 29.7 | 1058.5 | 7.33 | 207.38 | 458.5 | 191.4 |
| 1983 | 1534.7 | 1009.2 | 221.1 | 291.9 | 12.6 | 1095.5 | 7.9 | 215.34 | 509.2 | 201.72 |
| 1984 | 1639.9 | 1062.4 | 289.6 | 302.1 | -14.2 | 1169.1 | 8.62 | 223.44 | 547.3 | 209.09 |

1. Variables in billions of 1972 dollars, $r_{nom}$ in %, *P* and *w* are indices (1972=100).
2. Average interest rates on bonds issued by top level companies.
3. GNP deflator.
4. $M_1$ stock.
5. Average hourly nominal wage.

Source: W. J. Baumol and A. S. Blinder, *Economics – Principles and Policy*, 3rd ed. (N. York, Harcourt Brace Jovanovich, 1985), and Greene (1997).

## Appendix 2
## Financial data – annual rates of return: USA, 1950-90 (%)

| | Stocks Small cap.[1] | Stocks Large cap.[2] | T-Bonds Long run[3] | T-Bonds Intermed.[4] | T-Bills[5] | Inflation CPI |
|---|---|---|---|---|---|---|
| 1950 | 45.48 | 32.68 | -0.96 | 0.25 | 1.20 | 5.93 |
| 1951 | 9.41 | 3.47 | -1.95 | 0.36 | 1.49 | 6.00 |
| 1952 | 6.36 | 8.91 | 1.93 | 1.63 | 1.66 | 0.75 |
| 1953 | -5.68 | -1.74 | 3.83 | 3.63 | 1.82 | 0.75 |
| 1954 | 65.13 | 52.55 | 4.88 | 1.73 | 0.86 | -0.74 |
| 1955 | 21.84 | 31.44 | -1.34 | -0.52 | 1.57 | 0.37 |
| 1956 | 3.82 | 6.45 | -5.12 | -0.90 | 2.46 | 2.99 |
| 1957 | -15.03 | -11.14 | 9.46 | 7.84 | 3.14 | 2.90 |
| 1958 | 70.63 | 43.78 | -3.71 | -1.29 | 1.54 | 1.76 |
| 1959 | 17.82 | 12.95 | -3.55 | -1.26 | 2.95 | 1.73 |
| 1960 | -5.16 | 0.19 | 13.78 | 11.98 | 2.66 | 1.36 |
| 1961 | 30.48 | 27.63 | 0.19 | 2.23 | 2.13 | 0.67 |
| 1962 | -16.41 | -8.79 | 6.81 | 7.38 | 2.72 | 1.33 |
| 1963 | 12.20 | 22.63 | -0.49 | 1.79 | 3.12 | 1.64 |
| 1964 | 18.75 | 16.67 | 4.51 | 4.45 | 3.54 | 0.97 |
| 1965 | 37.67 | 12.50 | -0.27 | 1.27 | 3.94 | 1.92 |
| 1966 | -8.08 | -10.25 | 3.70 | 5.14 | 4.77 | 3.46 |
| 1967 | 103.39 | 24.11 | -7.41 | 0.16 | 4.24 | 3.04 |
| 1968 | 50.61 | 11.00 | -1.20 | 2.48 | 5.24 | 4.72 |
| 1969 | -32.27 | -8.33 | -6.52 | -2.10 | 6.59 | 6.20 |
| 1970 | -16.54 | 4.10 | 12.69 | 13.93 | 6.50 | 5.57 |
| 1971 | 18.44 | 14.17 | 17.47 | 8.71 | 4.34 | 3.27 |
| 1972 | -0.62 | 19.14 | 5.55 | 3.80 | 3.81 | 3.41 |
| 1973 | -40.54 | -14.70 | 1.40 | 2.90 | 6.91 | 8.71 |
| 1974 | -29.74 | -26.40 | 5.53 | 6.03 | 7.93 | 12.34 |
| 1975 | 69.54 | 37.26 | 8.50 | 6.79 | 5.80 | 6.94 |
| 1976 | 54.81 | 23.98 | 11.07 | 14.20 | 5.06 | 4.86 |
| 1977 | 22.02 | -7.26 | 0.90 | 1.12 | 5.10 | 6.70 |
| 1978 | 22.29 | 6.50 | -4.16 | 0.32 | 7.15 | 9.02 |
| 1979 | 43.99 | 18.77 | 9.02 | 4.29 | 10.45 | 13.29 |
| 1980 | 35.34 | 32.48 | 13.17 | 0.83 | 11.57 | 12.52 |
| 1981 | 7.79 | -4.98 | 3.61 | 6.09 | 14.95 | 8.92 |
| 1982 | 27.44 | 22.09 | 6.52 | 33.39 | 10.71 | 3.83 |
| 1983 | 34.49 | 22.37 | -0.53 | 5.44 | 8.85 | 3.79 |
| 1984 | -14.02 | 6.46 | 15.29 | 14.46 | 10.02 | 3.95 |
| 1985 | 28.21 | 32.00 | 32.68 | 23.65 | 7.83 | 3.80 |
| 1986 | 3.40 | 18.40 | 23.96 | 17.22 | 6.18 | 1.10 |
| 1987 | -13.95 | 5.34 | -2.65 | 1.68 | 5.50 | 4.43 |
| 1988 | 21.72 | 16.86 | 8.40 | 6.63 | 6.44 | 4.42 |
| 1989 | 8.37 | 31.34 | 19.49 | 14.82 | 8.32 | 4.65 |
| 1990 | -27.08 | -3.20 | 7.13 | 9.05 | 7.86 | 6.11 |

1. Russel 2000.
2. Standard & Poor’s 500.
3. At least 20 year maturity.
4. Seven year maturity.
5. Approximately 30 day maturity. It is assumed they are rolled over.

Source: Zvi Bodie, Alex Kane and Alan J. Marcus. *Essentials of Investments*, 5th ed. (N. York, McGraw−Hill, 2004).

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