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A macro-econometric model containing income distribution, price changes and Government financing: Solutions and dynamic analysis for the US

Manuel A. R. da Fonseca

Textos para Discussão

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A macro-econometric model containing income distribution, price changes and Government financing: Solutions and dynamic analysis for the US

Autor

Manuel A. R. da Fonseca *

* Programa de Pós-graduação Lato Sensu – MBA em Finanças (UFRJ)

Resumo:

O principal objetivo deste texto é avançar em relação a um artigo anterior sobre o uso de modelos macro-econométricos estruturais simples aplicados à economia dos EUA em um intervalo de 100 anos (da Fonseca, 2021). Esse artigo anterior incluía um modelo macro com relações baseadas na agregação de equações de equilíbrio intersetorial, que possibilitava estabelecer uma relação entre o produto total dos setores e os níveis de renda agregada, separados em lucros e salários, e de emprego. O modelo também incluía equações para o nível geral de preços, financiamento governamental e agregados monetários. Uma solução-base e uma simulação alternativa foram obtidas para este modelo, juntamente com uma análise de multiplicadores e um exercício simples de previsão.

Abstract:

The main objective of this text is to advance in relation to a previous article on the use of simple structural macro-econometric models applied to the US economy over a 100-year interval (da Fonseca, 2021). That earlier paper included a macro-model with relations based on the aggregation of intersectoral equilibrium equations, which made possible to establish a relationship between the total product of the sectors and the levels of aggregate income, separated into profits and salaries, and employment. The model also included equations for the general price level, Government financing and major monetary aggregates. Base and alternative solutions are derived for this model, together with a multiplier analysis and a very simple forecasting exercise.

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(Revised October 2022)

Manuel A. R. da Fonseca
MBA Program in Finance
Rio de Janeiro – UFRJ

Introduction

The main objective of this text is to advance in relation to a previous article on the use of simple structural macro-econometric models applied to the US economy over a 100-year interval (da Fonseca, 2021). That earlier paper included a macro-model with relations based on the aggregation of intersectoral equilibrium equations, which made possible to establish a relationship between the total product of the sectors and the levels of aggregate income, separated into profits and salaries, and employment. The model also included equations for the general price level, Government financing and major monetary aggregates.

In Section 1, the essentials of the macro-econometric model – including equations, variables, and units of measurement – are presented. The data series and parameter estimations are described in the following Section. Model simulations for a base and alternative solution are included in Section 3, which also contains a multiplier analysis for the system of equations, and a simple exercise in forecasting for five years in the future.

1. Main characteristics of the model

The model contains four production sectors – agriculture (including forestry, fishing, and hunting), manufacture (with utilities and construction), trade and transportation, and services. The outputs of these sectors are determined directly by income levels generated in the production process – that is, the consumption function is not included explicitly. Moreover, private investment is also internalized in the system of equations, but it is important to keep in mind that production in the manufacturing sector (Y_{MNF}), has a dynamic trajectory almost identical to private investment. Therefore, to a large extent, including that variable is equivalent to include investment. Table 1 contains the equations of the model.

Table 1. Equations of the macro-econometric model with income distribution, price changes and Government financing

Equilibrium output and aggregate production function	1. $Y = Y_{AGR} + Y_{MNF} + Y_{TRD} + Y_{SRV} + Govt\ Sector$ 2. $L = a_{w1}\ Y_{AGR} + a_{w2}\ Y_{MNF} + a_{w3}\ Y_{TRD} + a_{w4}\ Y_{SRV}$
Effective demand and the generation of income	3. $Y_{MNF} = b_0 + b_1\ Y_{-1} + b_2\ \Delta\ (\Pi/P - T_\Pi) + b_3\ \Delta\ G + b_4\ r + b_5\ Dummy + \varepsilon_3$ 4. $Y_{TRD} = b_0 + b_1\ Y_{-1} + b_2\ \Delta\ (wL/P - Tw) + b_3\ \Delta\ G + b_4\ r + \varepsilon_4$ 5. $Y_{SRV} = b_0 + b_1\ Y_{-1} + b_2\ \Delta\ (wL/P - Tw) + b_3\ r + \varepsilon_5$ 6. $r = b_0 + b_1\ M/P + b_2\ \Delta\ Y + \varepsilon$ 7. $wL/P = (w\ L\ Kw)\ /\ P$ 8. $\Pi/P = Y - IndTax - wL/P$
Determination of nominal variables and price changes	9. $M = \mu\ B$ 10. $B = \alpha\ (B + Debt)$ 11. $B + Debt = (B + Debt)_{-1} + G\ P + Interest + Subsids - T\ P$ 12. $Interest = \left[\left(\frac{P}{P_{-1}}\right)\left(1 + \frac{r}{100}\right) - 1\right] Debt_{-1}$ 13. $Debt = (1 - \alpha)\ (B + Debt)$ 14. $\ln P = \ln P_{-1} + b_1\ [\Delta \ln (M\ V) - \Delta \ln Y] + b_2\ \Delta \ln w + b_3\ \Delta \ln Inputs + b_4\ \Delta \ln e + \varepsilon_{14}$ 15. $\ln w = \ln w_{-1} + b_1\ \Delta \ln P_{-1} + \varepsilon_{15}$

Description of the variables (all variables are in billions of 2012 dollars, except if stated otherwise).

A. Endogenous variables.

Y	Gross National Product;
L	Total employment (thousands of individuals);
Y_{MNF}	Real value added in manufacture, utilities and construction;
Y_{TRD}	Real value added in trade and transportation;
Y_{SRV}	Real value added in services;
r	Interest rate on corporate bonds (Moody's Aaa rating), deflated by GDP price index (%);
wL/P	Total compensation of employees;
Π/P	Total gross operating surplus;
M	M1 (billions of dollars in December);
B	Monetary base (billions of dollars in December);
Interest	Federal Government interest payments (billions of dollars in December);
Debt	Total federal debt (billions of dollars in December);
P	GDP price index (2012 = 100);
w	Average earnings of non-supervisory employees (dollars per hour).

B. Exogenous variables and varying parameters.

Y_{AGR} Real value added in agriculture, forestry, fishing, and hunting;

G Government purchases;

EX-IM Net exports;

T Real taxes;

IndTax Tax on products less subsidies

Subsids Federal Government subsidies (billions of dollars);

Govt sector Real value added by the Government sector;

Inputs All commodities, Producer Price Index;

e Trade weighted U.S. dollar index (1973=100);

a_w Sectoral labor coefficient;

Kw Multiple that relates average income to wage per hour;

μ M1 multiplier;

α Share of monetary base on the sum (base + debt);

V M1 income velocity.

Dummy Equal to 1 in 2008, 2009, 2010 and 2020.

2. Data series and parameter estimations

Two main types of series were included in the model's data base, namely, standard macroeconomic data – series from St. Louis Fed (https://fred.stlouisfed.org) –, and industry data, including employment in the production sectors – series from the Bureau of Economic Analysis, BEA's Industry Accounts (www.bea.gov). The various units of measurement are displayed in Table 2.

Table 2. Units of measurement in the series.

Billions of dollars, nominal values.
Billions of dollars – values in December, nominal values (monetary aggregates, federal debt).
Billions of 2012 dollars, real or constant values.
Price index, 2012=100 (GDP).
Other price indices.
Dollars per hour – average earnings of non-supervisory employees, nominal or constant (wages and salaries).
Thousands of individuals (labor force).

2.1. Stochastic equations in the model

The stochastic equations are 3, 4, 5 (value added in sectors, constant values), 6 (real interest rate, %), 14 (general price index), and 15 (average nominal wage). All equations were estimated using OLS. With the exception of equations 14 and 15, estimations based on the 2SLS method were also computed, but these latter estimates did not improve the accuracy of the model solutions – in effect, in the case of some variables, the model's accuracy was significantly worsened. Therefore, the much simpler OLS estimation procedure was adopted. Based on a non-rigorous evaluation, OLS seemed to provide simulation results that performed better in terms of model dynamics when compared with data available for the endogenous variables.

Table 3. OLS estimation results.

Sample interval: 1981-2019 (39 observations).				
Eq. 3 (Y_{MNF})				
Variable	Coeff.	St. Error	t-Stat.	Probab.
Const.	953,9221	307,2765	3,104443	0,003897
Y_{-1}	0,123097	0,015282	8,055131	2,69913E-09
$\Delta\ (\Pi/P - T_{\Pi})$	0,360634	0,460647	0,782886	0,439277
$\Delta\ G$	0,954142	0,518005	1,841956	0,074484
r	-31,3547	30,23720	-1,036958	0,307295
Dummy	-72,2944	90,38426	-0,799856	0,429514
R-Squared	0,940463			
Adj. R-Squared	0,931442			
S. E. of regression	136,7151			
Sum squared resid.	616803,8			
Eq. 4 (Y_{TRD})				
Variable	Coeff.	St. Error	t-Stat.	Probab.
Const	-89,3068	235,6763	-0,37894	0,707090
Y_{-1}	0,158804	0,011505	13,80305	1,69903E-15
$\Delta\ (wL/P - Tw)$	0,293883	0,170091	1,727799	0,093099
$\Delta\ G$	0,519263	0,415345	1,250197	0,219764
r	-20,7828	23,64464	-0,87896	0,385590
R-Squared	0,972884			
Adj. R-Squared	0,969693			
S. E. of regression	110,9749			
Sum squared resid.	418724,9			
Eq. 5 (Y_{SRV})				
Variable	Coeff.	St. Error	t-Stat.	Probab.
Const	-907,135	292,0074	-3,10655	0,003742
Y_{-1}	0,565753	0,014366	39,38086	1,40598E-30
$\Delta\ (wL/P - Tw)$	0,357290	0,225622	1,583580	0,122285
r	-3,97368	27,51646	-0,144411	0,886004
R-Squared	0,995570			
Adj. R-Squared	0,995190			
S. E. of regression	147,5596			
Sum squared resid.	762084,5			
Sample interval: 1971-2020 (50 observations).				
Eq. 6 (r)				
Variable	Coeff.	St. Error	t-Stat.	Probab.
Const	5,061974	0,779128	6,496973	4,74462E-08
M/P	-0,00094	0,000377	-2,50425	0,015799
$\Delta\ Y$	0,001371	0,001236	1,109525	0,272848
R-Squared	0,138458			
Adj. R-Squared	0,101796			

S. E. of regression	2,035205			
Sum squared resid.	194,6769			

Sample interval: 1971-2019 (49 observations).
Eq. 14 (ln P/P_{-1})

Variable	Coeff.	St. Error	t-Stat.	Probab.
$\Delta \ln (MV) - \Delta \ln Y$	0,385347	0,060539	6,365257	8,94437E-08
$\Delta \ln w$	0,438785	0,057874	7,581683	1,41004E-09
$\Delta \ln Inputs$	0,098343	0,027328	3,598624	0,000793
$\Delta \ln e$	0,008287	0,017483	0,474028	0,637772
S. E. of regression	0,007038			
Sum squared resid.	0,002229			

Sample interval: 1972-2020 (49 observations).
Eq. 15 (ln w/w_{-1})

Variable	Coeff.	St. Error	t-Stat.	Probab.
$\Delta \ln P_{-1}$	1,012455	0,050172	20,17981	4,31126E-25
S. E. of regression	0,014197			
Sum squared resid.	0,009675			

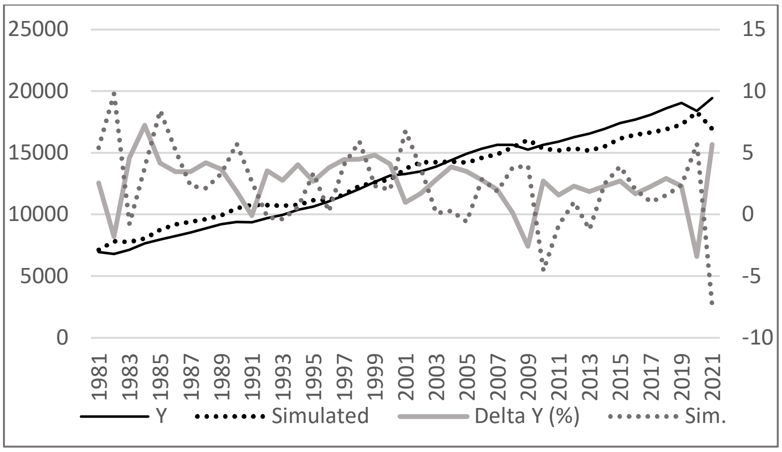
There are two additional equations (16 and 17) for total wages and total profits net of taxes. In the construction of these series, the tax total was proportionately divided into the two income segments.

One major issue with data for monetary variables was the change in the definition of M1 adopted by the Fed in May 2020, which caused a fourfold increase in the value for that month. To deal with this change, an adjustment has been made to bring recent data in line with the original series.

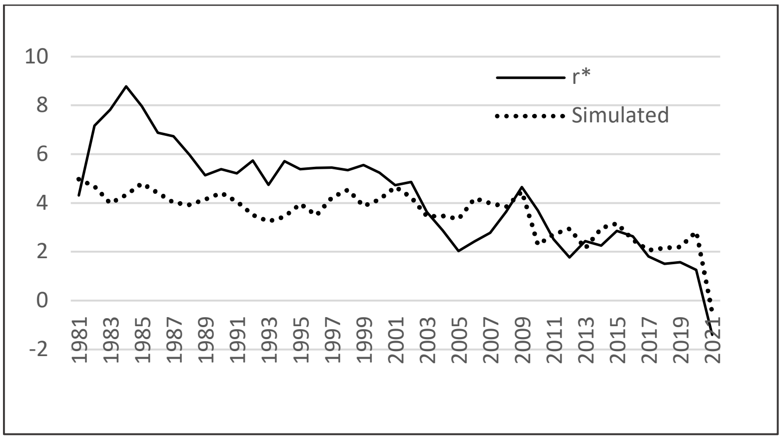
3. Solution of the model

Two prominent methods of solution for nonlinear systems of equations were applied to this model. The simplest alternative, function iteration, does not use derivatives. It applies eq. (12) in da Fonseca (2021) directly to the nonlinear system. The starting period was 1981 – that is, only data up to 1980 were used for the endogenous variables.

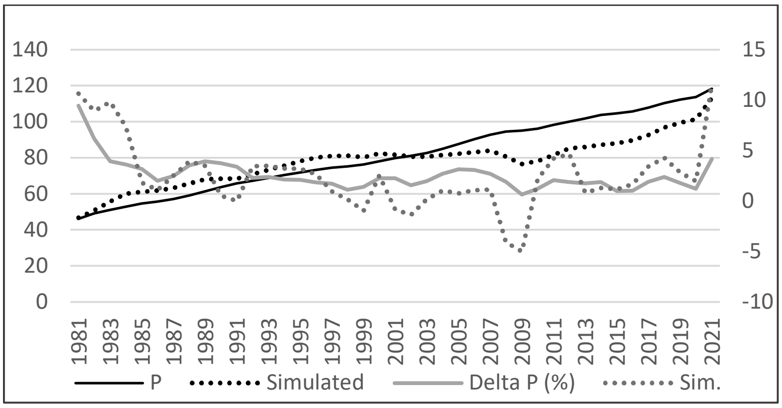
The so-called Newton method, which requires the construction of the Jacobian matrix, was also used – in the current version, this matrix has dimension 17x17 and, although most entries contain zeros, its construction does require some effort. In this procedure, eq. (13) in da Fonseca (2021) is used, and the Jacobian matrices were constructed for some specified periods, separated by intervals of ten years (approximately). The Jacobian matrix can also be used to provide first-derivative (multiplier) solutions. Historical and simulated values for some endogenous variables are included in Graphs 1 to 4. Graph 5 contains data for three exogenous variables for a longer period – since 1971.



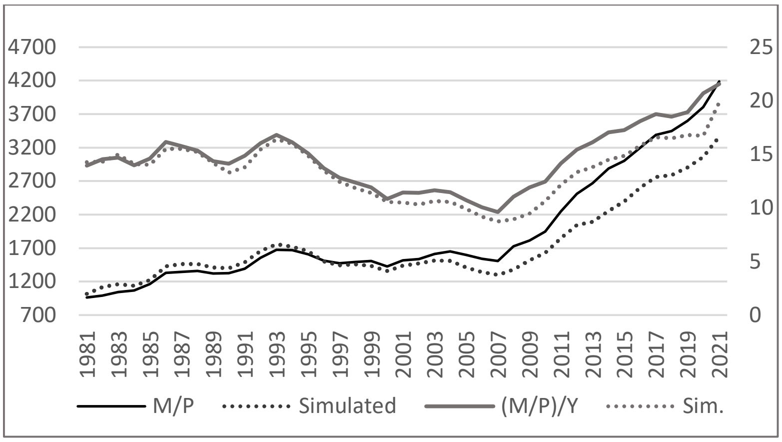
Graph 1. Y, Gross National Product, 1981-2021 – historical values and solutions (billions of 2012 dollars and rates of change, %).



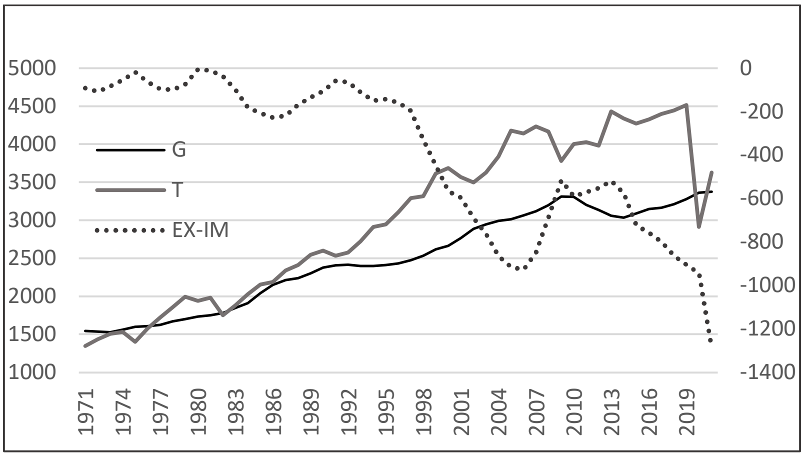
Graph 2. r, Interest rate on corporate bonds deflated by GDP price index, 1981-2021 – historical values and solutions (%).



Graph 3. P, GDP price index, 1981-2021 – historical values and solutions (2012 = 100 and rates of change, %).



Graph 4. M1, 1981-2021 – historical values and solutions (billions of dollars in December deflated by GDP price index and share of GDP, %).



Graph 5. Exogenous variables, G, T and EX–IM, 1971-2021 – historical values (billions of 2012 dollars).

Based on Graphs 1 to 5, some initial conclusions can be derived. One of the most important is that the model is not able to simulate the sequence of periods of stronger expansion and recession that took place in the forty years since the early 1980s (Graph 1). From 1981 to 1993, it can be argued that the model followed the macroeconomic cycles with a one-period delay. For the last three decades, however, the solutions, for the most part, carried the wrong signs – that is, growth when there really was recession and the other way around.

The observation in the previous paragraph is in clear opposition to the results obtained for an earlier period (1975-84). More specifically, a model based on Keynesian analysis was able to simulate with surprisingly accuracy the main macroeconomic variables in the US for the thirty years initiated in 1975, which was the first solution period (da Fonseca, 2021). Even the trajectory of GDP and aggregate investment in the first half of the 1980s was accurately simulated – which, in the case of the first variable, did not occur at all for the model examined in this text, whose solution started in 1981.

Significant recessions were observed in the last four decades, in cycles of about ten years: 1981-82, 1990-91, 2001, and 2008-9. As a rule, the model was not successful in simulating the change of economic activity in those years. In the case of the recessions that had a stronger financial component, this shortcoming might be expected – in fact, it would be quite unlikely that a structural model, centered on outputs of sectors and income distribution, could reproduce the financial crisis of 2008-9. In relation to this point, one important exception is the simulation of variable P (GDP price index, Graph 3). To a certain extent, the model was indeed successful in describing the trajectory of the average price level, both in recession and growth years (with some overshooting).

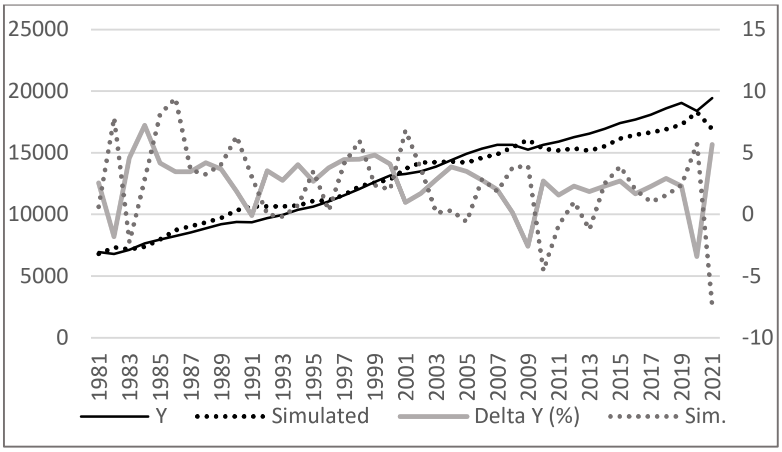
A key conclusion, then, is that the post-World War II period, until the mid-1980s, can be characterized as predominantly consistent with the macroeconomic analysis based on Keynes and Kalecki. But the same does not apply in subsequent decades, when movements derived mainly from financial trends apparently predominated.

An additional observation in relation to the model's solutions applies to the behavior of exogenous variables in 2020-21 – the period most strongly affected by the covid-19 epidemic. As can be seen in Graph 5, there was a brutal reduction in tax collection, accompanied by a sharp deterioration in the trade balance. In terms of aggregate demand, these trends have opposite effects. However, the initial model solution showed a very strong growth in 2020 due to the sharp increase in disposable income. Therefore, it was necessary to use a procedure (through the dummy variable in eq. 3) to reduce GDP growth in 2020. In relation to the value of variable T in 2020, it should be mentioned that the series of taxes in real values is obtained by the difference between Gross National Product (Y) and Disposable Aggregate Income (not used directly in the model). The steep fall in the value for 2020 is associated with an increase in subsidies – compared to 2019, the value was multiplied by 10 –, and in transfers to households (80% increase).

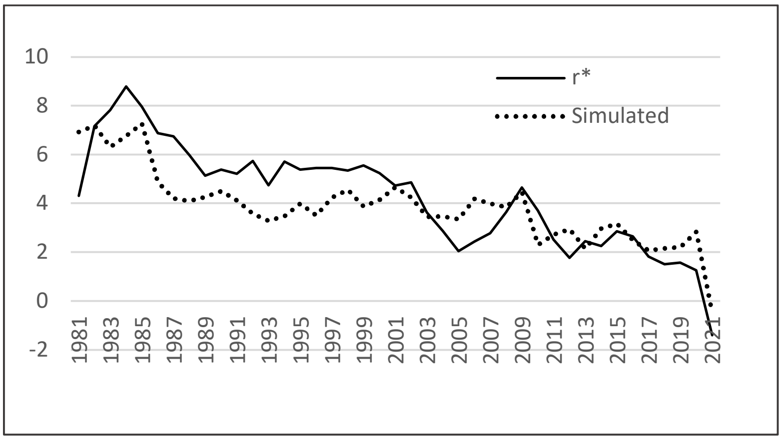
3.1. Simulation of r (real interest rate) in the first half of the 1980s

An exercise was conducted to evaluate the performance of the model in a period marked by a major change in the Fed's interest rate policy. In particular, the trajectory of interest rates in the first half of the 1980s was examined in more detail. As can be seen in Graph 2, the initial solution of the model did not reproduce the strong upward movement of interest rates between 1981 and 1989. An adjustment was then made on eq. (6) so that the simulated values were closer to the actual data. Thus, one can evaluate what the model suggests in terms of the effects of interest rates on main macroeconomic variables, and, in particular, on inflation. The simulations for some variables obtained with this secondary solution are represented in Graphs 6 to 8.

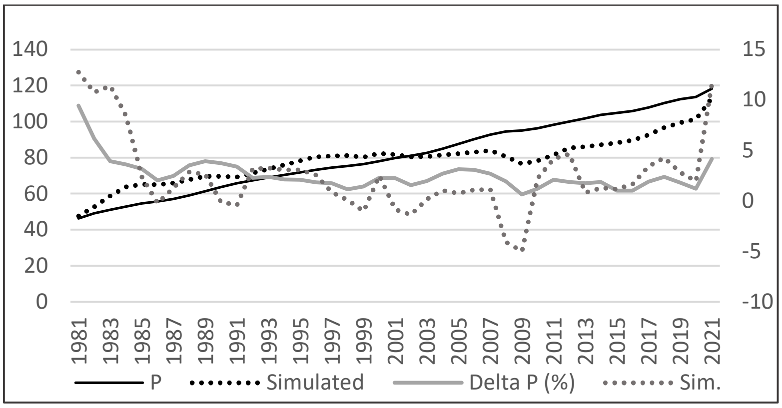
First, it should be clear that, in terms of this model, there is no direct relation between the interest rate (r) and the price level (P). There are, however, two important indirect relationships, that is, through aggregate production (Y) and due to the effect of interest rates on the exchange rate (variable e) – the latter, however, is an exogenous variable and, thus, this effect cannot be evaluated through this model.



Graph 6. Y, Gross National Product, 1981-2021 – modified solution.



Graph 7. r, Interest rate on corporate bonds deflated by GDP price index, 1981-2021 – modified solution.

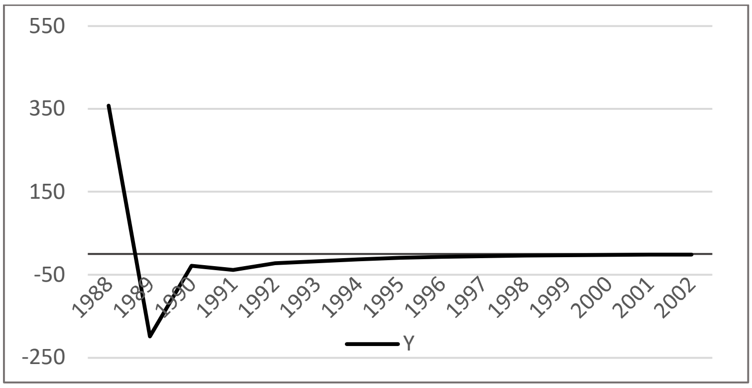


Graph 8. P, GDP price index, 1981-2021 – modified solution.

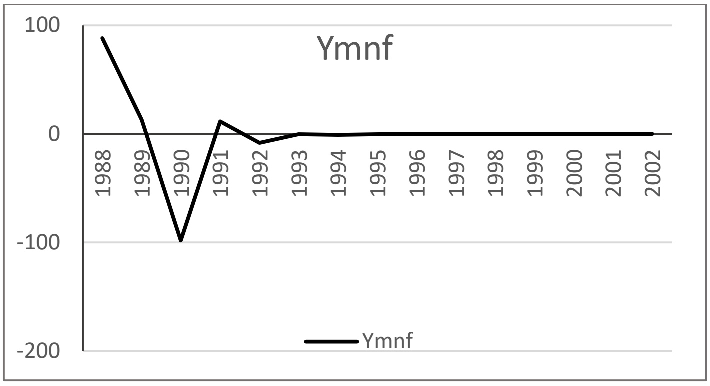
A comparison of Graphs 3 and 8 reveals that, given inflationary pressures resulting from stronger money supply, and higher wages and commodity prices, the model indicates that a reduction of output growth caused by higher interest rates contributes to <u>increase</u> the inflation rate – and not the opposite, as it is the predominant view of economists in central banks around the world. Nevertheless, Graph 8 also shows that there is a steeper decrease of inflation in 1986, when compared to the initial solution (Graph 3).

3.2. Multiplier analysis: Effects of a one-time increase in G (Government purchases)

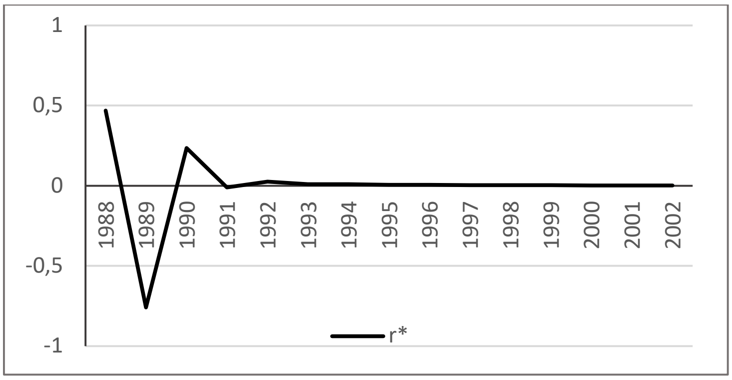
To evaluate the dynamic multipliers of this model, a simulation exercise was carried out that consisted of an increase of $100 (billion) in government spending in a given period. The year selected was 1988, and this value corresponds to an increase of approximately 5%. Graphs 9 through 15 contain, for some endogenous variables, the differences between the values in this multiplier-based simulation and the original solution. The multiplier results are, in general terms, consistent with the one developed for the Klein model based on Kalecki's macroeconomic theory (Theil and Boot, 1962). An interesting finding is that the multipliers for variable Y_{MNF} (a proxy for aggregate private investment) indicate that the model generates a dynamic pattern similar to a crowding-out effect, since, two years after the increase in G, there is a decrease in that variable of the same magnitude as the original upward movement (Graph 10).



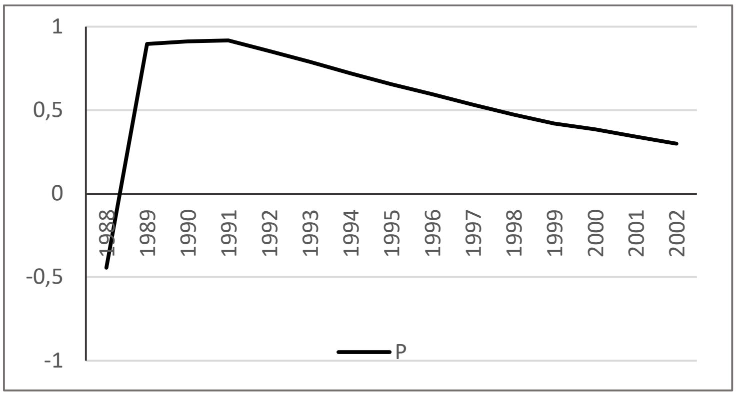
Graph 9. Y, Gross National Product – changes in relation to base solution.



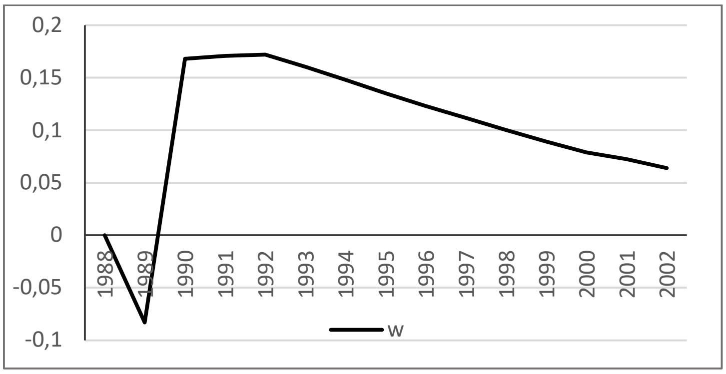
Graph 10. Y_{MNF}, Real value added in manufacture – changes in relation to base solution.



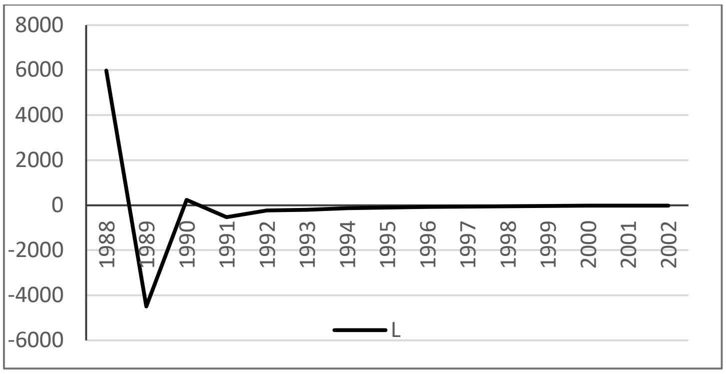
Graph 11. r, Interest rate deflated by GDP price index – changes in relation to base solution.



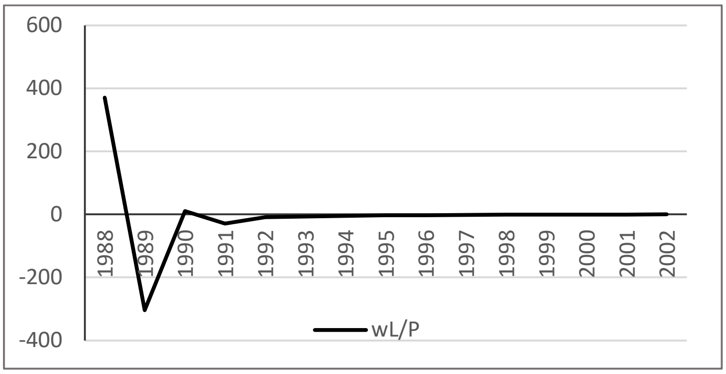
Graph 12. P, GDP price index – changes in relation to base solution.



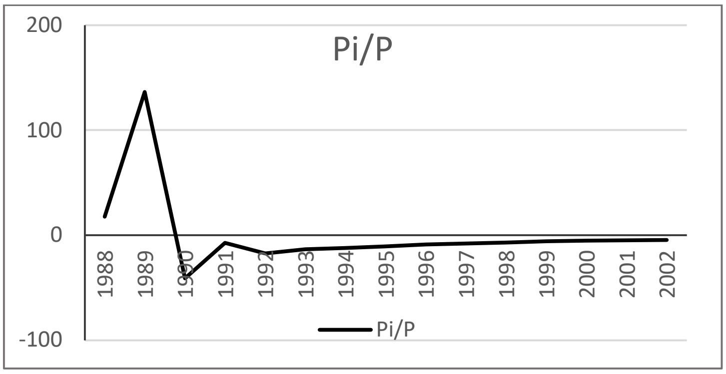
Graph 13. w, Average earnings of employees – changes in relation to base solution.



Graph 14. L, Total employment – changes in relation to base solution.



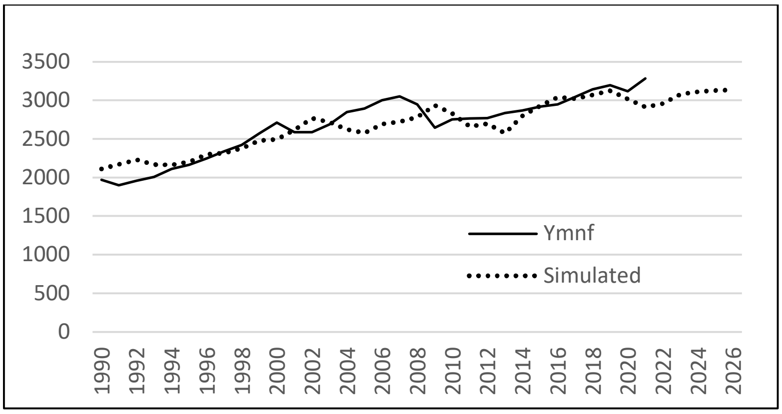
Graph 15. wL/P, Total compensation of employees – changes in relation to base solution.



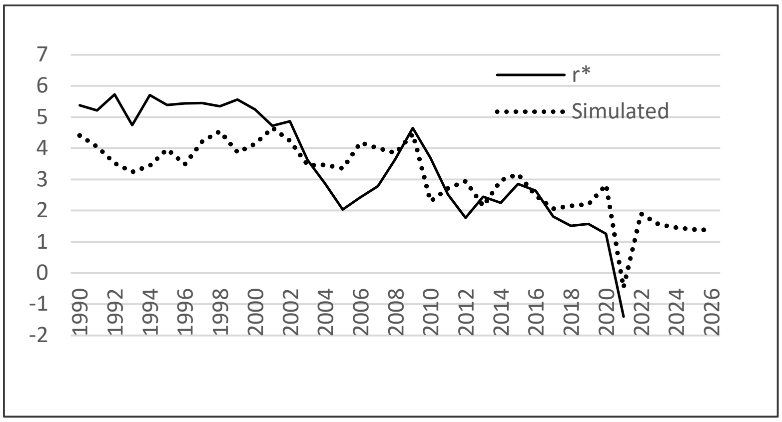
Graph 16. Π/P, Total gross operating surplus – changes in relation to base solution.

3.3. Simulations for five years beyond 2021

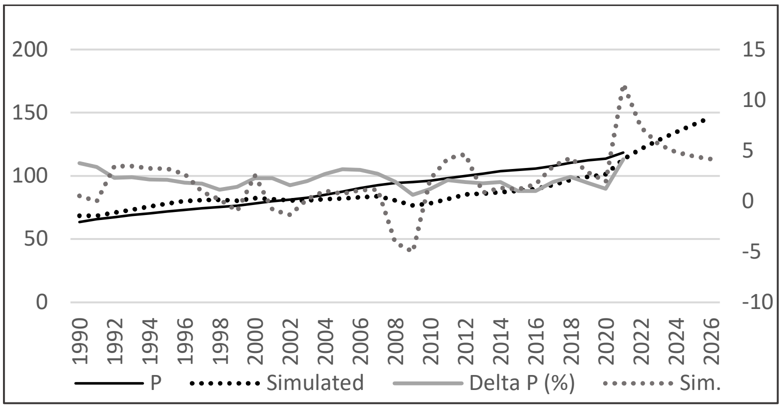
A simple exercise was performed to provide a glimpse of the model as a forecasting tool. In this enterprise, there was no tentative to outline alternative scenarios for the exogenous variables, instead all exogenous elements were kept at their 2021 values. Therefore, in this rudimentary forecasting, there is a clear bias towards stagnation, when considering GDP and sectoral productions, and also a tendency to underestimate inflationary pressures. The results for a few variables are included in Graphs 17 to 20.



Graph 17. Y_{MNF}, Real value added in manufacture – 1990-2026.



Graph 18. r, Interest rate deflated by GDP price index – 1990-2026.



Graph 19. P, GDP price index – 1990-2026.

4. Concluding remarks

In the forty-year period analyzed in previous Sections, the cycles of economic expansion and recession were not captured by the model dynamics. One explanation is that these patterns do not result from major macroeconomic relations, centered on the circular effects of aggregate demand and supply, income generation, and changes in general price levels. There is a possibility, then, that the causal factors of the economic cycles in this period are predominantly of a financial nature – this assertion certainly applies to the crisis initiated in 2007-8.

In relation to the 1982 crisis, it is associated to the strong increase of interest rates promoted by the Fed to tackle inflation. Such increase is not reflected in the model's base solution (Graphs 1 to 4). As an alternative procedure, a modification was introduced in eq. 5 for the 1981-85 period so that the simulated values for the real interest rate could get closer to historical values. This alternative solution is illustrated in Graphs 6 to 8, and it provides a perspective, based on the model, of the effects of interest rates on main macroeconomic variables

Moreover, a multiplier analysis was developed through a simulation based on an increase of 5% in Government spending in a given year. The results, which are generally consistent with those obtained for a reference model, are included in Graphs 9 to 16. Lastly, a very simple forecasting exercise was implemented without attempting to provide alternative scenarios for the exogenous variables – all exogenous elements were kept at their 2021 values instead.

5. References

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