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A Simple Approach to Assess if a Financial "Bubble" is Present: The Case of Bitcoin

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Textos para Discussão

No. 6 – setembro, 2018.

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Título

A Simple Approach to Assess if a Financial "Bubble" is Present: The Case of Bitcoin

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Resumo: O objetivo deste artigo é avaliar se o comportamento recente do preço do Bitcoin pode ser caracterizado como uma "bolha" do mercado financeiro. Para realizar essa avaliação, adotamos uma definição estatística de uma "bolha" derivada da hipótese do mercado eficiente, e propomos um método simples para testar essa proposição, baseado no modelo de séries temporais conhecido como passeio aleatório (random walk). Analisamos os dados disponíveis para os preços do Bitcoin, juntamente com um ativo selecionado como benchmark, e realizamos testes estatísticos derivados de equações simples de regressão. A principal conclusão é que há evidência clara de que o Bitcoin segue o padrão de uma "bolha" financeira – pelo menos, esse padrão é muito mais evidente no caso do Bitcoin do que no índice de ações usado como referência.

Palavras-chave: Bitcoin, "Bolhas" no mercado financeiro, Séries temporais, Análise estatística de regressão.

Abstract: This paper's goal is to evaluate if the recent price behavior of Bitcoin can be characterized as a financial market "bubble". To deal with this assessment, we adopt a statistical definition of a "bubble" derived from the efficient market hypothesis and we propose a simple method to test this proposition, based on the time-series model known as random walk. We analyze the data available for Bitcoin prices, together with an asset selected as benchmark, and perform statistical tests derived from simple regression equations. The main conclusion is that there is consistent evidence that that Bitcoin follows the pattern of a financial "bubble" – at least, such pattern is more evident in the case of Bitcoin than in the stock index used as benchmark.

Key-words: Bitcoin, "Bubbles" in financial markets, Time series, Statistical regression analysis.

A Simple Approach to Assess if a Financial "Bubble" is Present: The Case of Bitcoin

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1. Introduction

What is Bitcoin? This question has recently gained increasing interest with the astonishing gains in value of this "created-by-technology" financial asset. In fact, several answers can be offered: It is a secretive (crypto) currency, a new and revolutionary commodity, an original and strikingly different "model of trust", a currency resulting from a decentralized network of qualified participants, a convenient form of payment based on new digital technologies, and so on…

Despite the diverse explanations for the Bitcoin phenomenon, many analysts appear to share the view that, whatever this new asset really is, it is most certainly than not just another case of a financial "bubble" – and, of course, one that may burst at any moment.[i] The main goal of this paper is, then, to evaluate this proposition – i.e. that the recent price behavior of Bitcoin can be characterized as a "bubble" in a financial market.

In order to deal with this assessment, we adopt a technical definition of "bubble" derived from the efficient market hypothesis (Section 2), and we propose a simple method to test this proposition, based on the time-series model best known as random walk (Section 3). Finally, we analyze the data available for Bitcoin prices, together with an asset selected as benchmark, and perform statistical tests derived from simple regression equations (Section 4).

The results obtained indicate that there is evidence that Bitcoin is indeed a recent case of a financial "bubble" – at least, there is evidence that it is more so than a benchmark stock index for small-capitalization companies.

1.1. Review of the Literature

Baur, Hong and Lee (2017) analyze if Bitcoin is a currency or an asset and claim that transaction data shows that Bitcoins are mainly used as a speculative form of investment. On the other hand, Glaser et al. (2014) question the uses of bitcoin for other purposes besides financial investment. The authors claim to have found strong evidence that users do not view bitcoin as a transaction facilitator but mainly as a speculative investment opportunity. BIS (2018) mentions pitfalls and risks to cryptocurrencies and its holders. It affirms that the required trust on a cryptocurrency – that is by nature unredeemable – can quickly dissipate because of the decentralized aspect of it. Another complication of decentralization is the lack a central issuer working to minimize volatility.

White, Marinakis, and Walsh (2018) select arguably acceptable cases of bubbles to compare with the case of bitcoin. The authors show that bitcoin exceeds in price appreciation all the selected cases, cautioning that, from a historical perspective, the rapid appreciation of bitcoin has shown to

be unsustainable. Gervais et al. (2014) elaborate that the decentralization – which at first is one of the selling points of Bitcoin – brings unknown risks. The authors found that more than 75% of Bitcoin mining power is controlled by only 6 pools. This goes against the original idea that decentralized operation and transparency would generate security and trust. In relation to problems not directly linked to decentralization, Moore and Christin (2012) examine 40 Bitcoin exchanges and find that 18 had been closed, with customer account balances often lost. From a different perspective, Brezo and Bringas (2012) list manners in that Bitcoin can be used in unlawful situations, pointing out that it lends itself to money laundering activities.

2. Definitions of Financial "Bubble"

Although the idea that the market behavior of an asset may be considered a "bubble" is a common place in academic research as well as in the popular media, apparently there is not a generally accepted definition of a financial "bubble". It is quite common to find technical articles discussing price behaviors that can be characterized as "bubbles", together with several historical experiences, and nowhere a clear characterization of this type of phenomenon is provided. There are, nevertheless, exceptions to this rule and Contessi and Kerdnunvong (2015) explore different definitions that have been suggested – although, from a scientific perspective, they can very well be questioned.[ii]

Further, in a technical letter provided by the Chicago Fed, the following definition is advanced:

> "What are asset bubbles? In general, […] a bubble exists when the market price of an asset exceeds its price determined by fundamental factors by a significant amount for a prolonged period."[iii]

Clearly, in this case, the burden of defining a "bubble" is transferred to the effort of specifying what exactly is a "price determined by fundamental factors". In any case, the authors of this essay also suggest a comparison with asset prices that behave according to the efficient market hypothesis.

> "The efficient market hypothesis asserts that […] actual and fundamental prices are always the same, and bubbles cannot exist unless they are driven by irrational behavior or market rigidities […]"[iv]

That is, based on this argument, "bubbles" occur when prices do not behave according to the efficient market hypothesis, which establishes that asset prices – and, in particular, stock prices – change only in response to new and unpredictable information. Therefore, these price changes must also behave in an unpredictable fashion. More specifically, the essence of the proposition known as the "efficient market hypothesis" is that asset prices should follow a *random walk* – i.e. that price changes should be random and unpredictable.[v]

3. Random Walk and Statistical Tests: A Methodological Proposition

A time-series model in which the value of a variable in one period is equal to its value in the previous period plus a random error is called a random walk. Such a model can be represented by the following equation: [vi]

$$x_t = x_{t-1} + e_t \qquad (1)$$

The assumptions imposed on the random variables e_t are:

a. Zero mean: $E(e_t) = 0$;
b. Constant variance: $E(e_t^2) = \sigma^2$;
c. Uncorrelated errors: $E(e_t\, e_s) = 0$, if $t \neq s$.

It is not difficult to show that, as a consequence of these assumptions, the following result applies to the variance of x_t:

$$V(x_t) = (t - 1)\sigma^2 \qquad (2)$$

That is, as $t \to \infty$, this variance becomes infinitely large. To deal with this problem, a simple transformation is commonly used:

$$y_t = x_t - x_{t-1} = e_t \qquad (3)$$

Therefore, one approach that can be used to assess if the price behavior of an asset should be described as a "bubble" is to evaluate if the *first differences* of the daily prices follow the assumptions used for the random errors e_t and, in special, if these variables are uncorrelated.

Given the standard definitions of the population correlation coefficient, ρ, and its sample estimator, r, a statistical test is available to assess the hypothesis H_0: $\rho = 0$.[vii] However, in a more convenient approach, based on the relation between correlation and regression analyses, a simple regression equation can be used.[viii] Considering that the assumptions of regression analysis are valid, the following result can be demonstrated for the regression equation $Z = \alpha + \beta\, W + \varepsilon$:

$$Cov(W, Z) = \beta\, V(W) \qquad (4)$$

Therefore, to test if the correlation coefficient (and the covariance) between W and Z is zero, it is sufficient to test the hypothesis H_0: $\beta = 0$. In this case, the following regression equation is used:

$$y_t = \beta\, y_{t-1} + \varepsilon_t \qquad (5)$$

4. Empirical analysis and results

In this Section, the methodology outlined in Section 3 is applied to Bitcoin price data. However, in order to obtain more meaningful conclusions, the Bitcoin data sample is compared with a *benchmark* – that is, an asset that can be considered a *standard* in relation to which other assets could be compared. In Table 1, summary statistics are presented for some financial indicators in the US market.

Table 1. Financial indicators in the US – annual data, 1926-2001.
Rates of return, %.

	Stocks *Small Cap*[1]	Stocks *Large Cap*[2]	T-Bonds Long Maturity	T-Bonds Medium Maturity	T-Bills	Inflation CPI
Geometrical aver.	12.2	10.5	5.2	5.1	3.8	3.0
Sample average	18.3	12.4	5.5	5.3	3.8	3.1
Standard deviation	43.4	23.6	9.9	8.3	5.1	5.4
Minimum	-52.7	-45.5	-8.7	-5.8	-1.6	-10.2
Maximum	187.8	54.6	32.7	33.4	15.0	18.1
Number of periods	76	76	76	76	76	76

Notes: [1] Russell 2000. [2] S&P 500.
Source: Statistical indicators obtained by the authors from original data in Bodie, Kane and Marcus (2009).

According to Table 1, the index for stocks of smaller companies (Russell 2000) has shown, over the years, the greatest volatility among the main financial indicators available for the US market, and it was chosen as the benchmark for the analysis of Bitcoin price data – which is also a very risky financial asset.

4.1. Russell 2000 – Statistical Analysis

In this analysis, data since 2009 was used – the start of the period when Bitcoin became available. From the original data, first differences were calculated (Figure 1). The main statistical indicators for this data-set appear below.

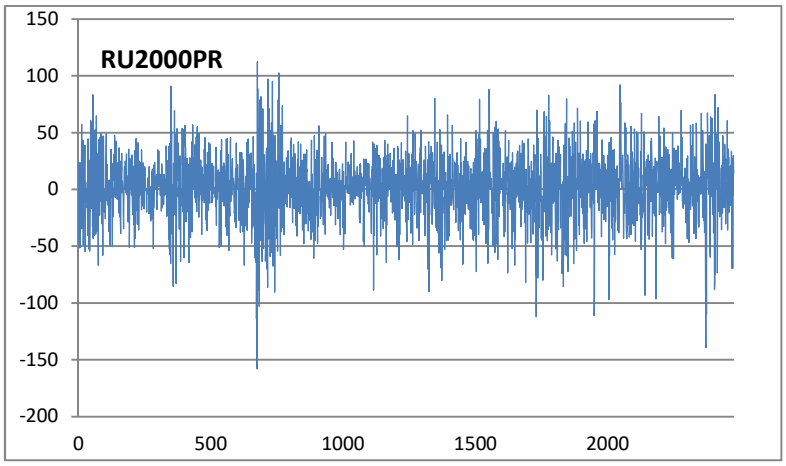

Figure 1. Russell 2000.
First differences calculated from the original daily data, 2009-2018.
Source: Obtained by the authors from original data in the Federal Reserve Bank of St. Louis. Available in https://fred.stlouisfed.org.

Table 2. Russell 2000 – Basic statistical indicators.
First differences calculated from the original daily data, 2009-2018.

1.305743	Average
0.604279	St. Error
2.7	Median
29.03686	St. Deviation
843.1391	Variance
-0.34955	Asymmetry
1.455473	Kurtosis
270.54	Range
-158.22	Minimum
112.32	Maximum
3014.96	Sum
2309	Count

Source: Obtained by the authors.

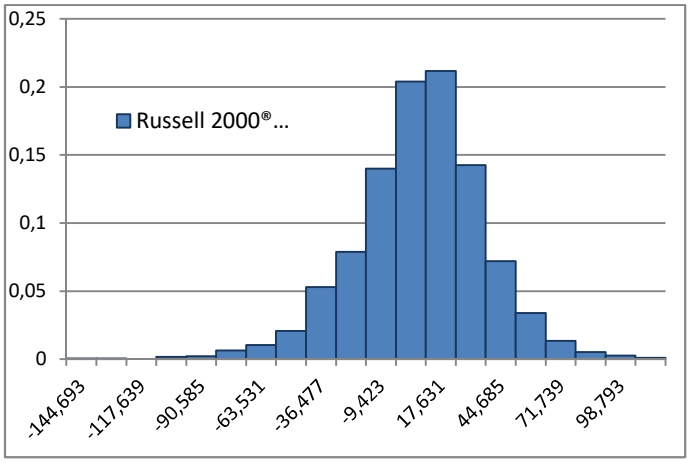

Figure 2. Russell 2000 – Histogram.
First differences obtained from the original daily data, 2009-2018.
Source: Obtained by the authors.

4.2. Bitcoin – Statistical Analysis

An initial examination of Bitcoin prices reveal a major structural change in the data set (Figure 3) – that is, considering the statistical properties, the latter period cannot be compared with the earlier one. As a result, the sample that was, in fact, used was initiated after the 1700th observation. Figure 4 depicts the data used.

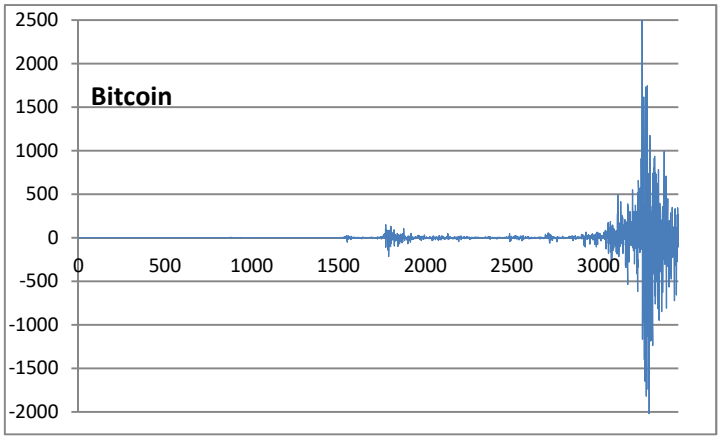

Figure 3. Bitcoin daily prices.
First differences calculated from the original data, 2009-2018.
Source: Obtained by the authors from original data available in https://charts.bitcoin.com/chart/price.

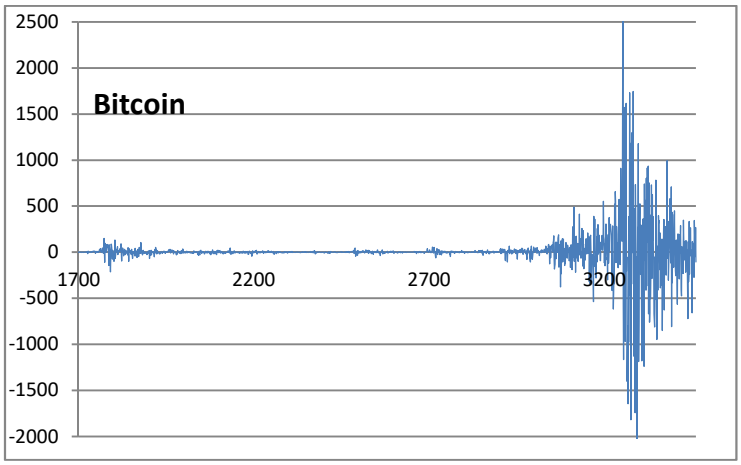

Figure 4. Bitcoin daily prices – Reduced sample.
First differences calculated from the original data, 2013-2018.
Source: Obtained by the authors.

As can be perceived from this Figure, the sample available for Bitcoin prices, even including only the latter period, still presents major structural changes that, in statistical regression analysis, is described as heteroskedastic regression errors. As it is well known (Verbeek, 2004) this problem in the errors of the regression equation invalidates most of the results and statistical tests. The alternative most often followed is to use the generalized least squares (GLS) estimator instead the more basic, ordinary least squares (OLS), version. Nevertheless, this alternative was not adopted at this stage of our research.

Table 3. Bitcoin – Basic statistical indicators.
First differences calculated from the original daily data, 2013-2018.

3.6115694	Average
5.4410889	St. Error
0.82	Median
228.5904999	St. Deviation
52253.6166	Variance
0.52252559	Asymmetry
36.0138685	Kurtosis
4843.54	Range
-2066.65	Minimum
2776.89	Maximum
6374.42	Sum
1765	Count

Source: Obtained by the authors.

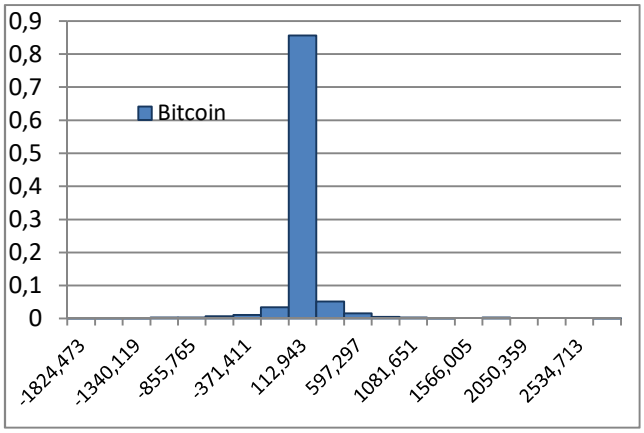

Figure 5. Bitcoin – Histogram.
First differences obtained from the original daily data, 2013-2018.
Source: Obtained by the authors.

4.3. A Test of Hypothesis – Is The Price Behavior a "Bubble"?

The main goal of this paper is to assess if the pattern of Bitcoin prices is in conformity with what most analysts describe as a financial "bubble". To test for this possibility, as outlined in Section 3, we evaluate if the asset's price pattern behaves as predicted by the efficient market hypothesis – that is, if the statistical properties of the data series is in accordance with the random walk time-series model. More specifically, we test if the correlation between first differences in sequential periods is zero. Further this test is set in the context of the statistical regression model. Therefore, in the regression equation (5) above, we test H_0: $\beta = 0$. The results obtained for Bitcoin prices, and for the Russell 2000 index, which is used as a benchmark, are shown below.

4.3.1. Russel 2000

The regression results for the benchmark are included in Table 4 and Figure 6.

Table 4. OLS, using observations 1-2476 (T = 2223)
Observations missing or incomplete were ignored: 253
Dependent variable: Russell

	Coefficient	*Standard Error*	*t-ratio*	*p-value*	
Russell_1	−0.0416852	0.0209984	−1.985	0.0472	**

Average dependent var.	1.343315	S.D. dependent var.	28.94991
Sum squared residuals	1862959	S.E. of regression	28.95541
R-square non-centered	0.001770	R- square centered	-0.000380
F(1, 2222)	3.940877	P-value(F)	0.047249

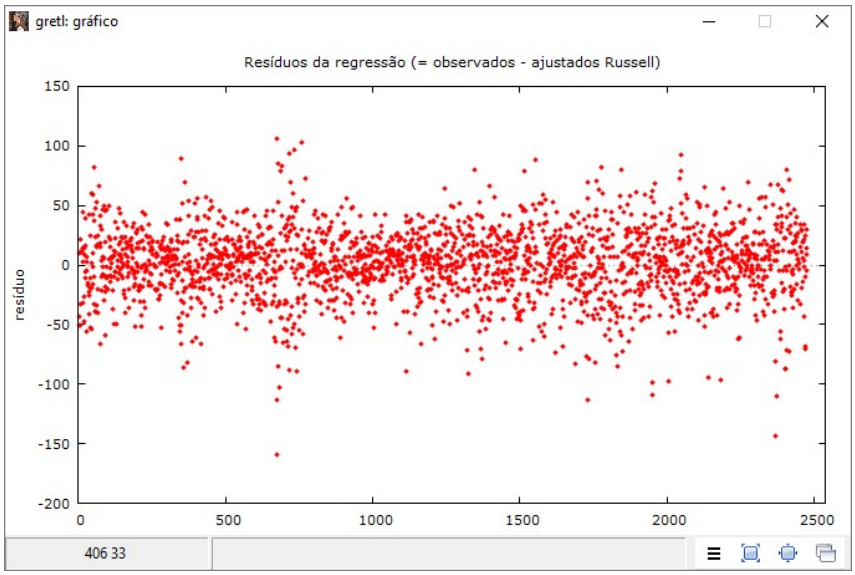

Figure 6. Residuals of regression in Table 4.

As we can see from Table 4, the hypothesis of zero correlation cannot be rejected only at the 1% significance level – but it can indeed be rejected at the 5% significance level. Therefore, according to the analysis presented in this paper, one can accept with 95% confidence the alternative hypothesis that, since 2009, the Russell 2000 behaves as a financial "bubble".

4.3.2. Bitcoin

The regression results for Bitcoin prices are included in Table 5 and Figure 7.

Table 5. OLS, using observations 1700-3462 (T = 1763)
Dependent variable: Bitcoin

	Coeficiente	*Erro Padrão*	*razão-t*	*p-valor*	
Bitcoin_1	0.0738327	0.0237595	3.108	0.0019	***

Average dependent var.	3.618712	S.D. dependent var.	228.7201
Sum squared residuals	91695832	S.E. of regression	228.1245
R-square non-centered	0.005451	R- square centered	0.005201
F(1, 1762)	9.656590	P-value(F)	0.001917

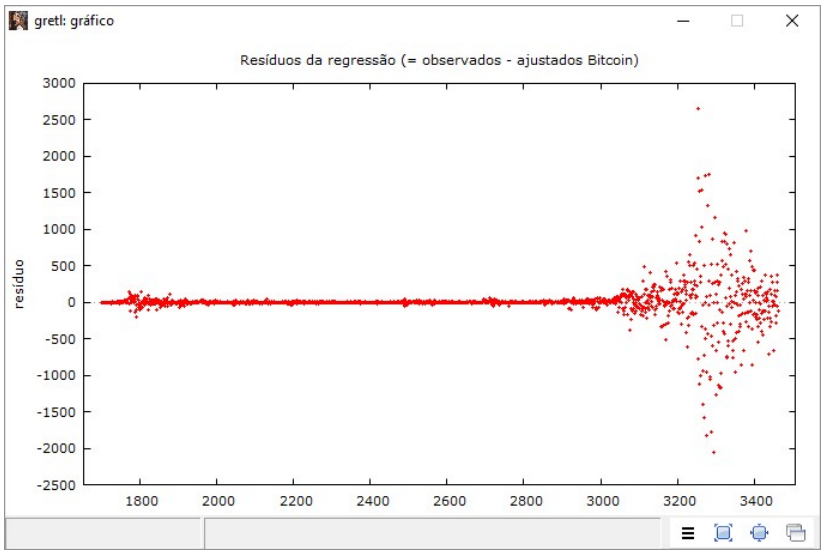

Figure 7. Residuals of regression in Table 5.

As we can see from Table 5, the hypothesis of zero correlation can be rejected at virtually any significance level. Therefore, according to the analysis presented in this paper, one can accept with virtually 100% confidence the alternative hypothesis that, since 2013, Bitcoin behaves as a financial "bubble". In other words, although this latter result is affected by the presence of heteroskedastic errors, the available data suggest that there is considerably more evidence of a financial "bubble" in the case of Bitcoin than in the Russell 2000 index.

4.3.3. A Further Comparative Analysis – The Case of Nasdaq

As it is well known, the trajectory of the Nasdaq in the late 1990s represents one of the clearest cases of a financial "bubble" in recent times – and that became known as the "dot-com" bubble. To bring an additional perspective to the analysis developed here, the regression analysis is also applied to the Nasdaq composite data. The results appear in Table 6. As can be perceived, the results for Nasdaq are located in an intermediate position, between the Russell 2000 and the bitcoin results.

Table 6. OLS, using observations 2-3129 (T = 3128)
Dependent variable: NASDAQCOM

	Coeficiente	*Erro Padrão*	*razão-t*	*p-valor*
NASDAQCOM_1	0.0397683	0.0178713	2.225	0.0261 **

Average dependent var.	0.583792	S.D. dependent var.	38.19060
Sum squared residuals	4554652	S.E. of regression	38.16486
R-square non-centered	0.001581	R- square centered	0.001348
F(1, 3127)	4.951770	P-value(F)	0.026135

5. Conclusion

In this paper, we propose a more precise, statistical definition to describe the situation when the price behavior of a financial asset should be characterized as a "bubble". According to the widespread view on the subject, a "bubble" exists in a financial market when the asset price exceeds its price determined by fundamental factors – by a significant amount and for a prolonged period. Moreover, the concept of a fundamental price is derived from the efficient market hypothesis – i.e., unless irrational behavior or market rigidities are present, the actual asset price coincides with the fundamental one.

From this basic propositions, we then use one of the main conclusions of the efficient market hypothesis as an indicator that an asset price that diverts from its fundamental price – i.e. that the dynamic path of the asset price follows the pattern of a random walk time series model. Next we perform a statistical test to evaluate if the correlation between sequential first differences calculated from the asset price is zero – hypothesis of uncorrelated errors. The version of the test that was used is based on the analysis of simple regression equations.

In Section 4, the statistical test to evaluate zero correlation was applied to Bitcoin prices and to an asset used as a benchmark – the Russell 2000 small-capitalization stock index. We conclude that, with close to 100% confidence, one can accept the alternative hypothesis of non-zero correlation – or, in other words, that since 2013 Bitcoin behaves as a financial "bubble". However, in the case of the Russel 2000 index, we infer that, with 95% confidence, the alternative hypothesis that this stock index behaves as a financial "bubble" should be accepted. Therefore, our main conclusion is that there is consistent evidence that that Bitcoin follows the pattern of a financial "bubble" – at least, such pattern is considerable more evident in the case of Bitcoin than in a benchmark stock index for small-capitalization companies.

Notes

[i] "[…] As bitcoin's value appreciates [… it] begins to look like a speculative bubble." (White, Marinakis and Walsh, 2018, p. 2).
[ii] For example, in one case, a "bubble" is defined as "an upward price movement over an extended range that then implodes." Thus, according to this definition, one could only ascertain the existence of a "bubble" after the fact – that is, after it burst.
[iii] Evanoff, Kaufman, and Malliaris (2012), p. 1.
[iv] Ibid.
[v] Bodie, Kane and Marcus (2009).
[vi] Shumway and Stoffer (2006). This is a especial case of the first order autoregressive, or AR(1), process $x_t = \delta + \theta\, x_{t-1} + e_t$.
[vii] Newbold (1984).
[viii] This approach is suggested in Wonnacott and Wonnacott (1972). It was used in Fonseca (2013).

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