Grupo de Pesquisa em
**Gestão e Planejamento Econômico-Financeiro**
**Universidade Federal do Rio de Janeiro – UFRJ**

# A hundred-year overview of the US economy: An appraisal from the perspective of simple macro-econometric models

Manuel A. R. da Fonseca

**Textos para Discussão**

**No. 9 – out. 2021.**

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**Título**

# A hundred-year overview of the US economy: An appraisal from the perspective of simple macro-econometric models

**Autor**

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**Resumo:**

Este texto tem como principal objetivo utilizar modelos macro-econométricos simples construídos para os EUA, e as respetivas bases de dados, para perseguir um objetivo no contexto de história económica. A questão aqui abordada é: Que informações e conclusões relevantes podem ser obtidas da análise destes modelos e, em particular, dos padrões dinâmicos das suas variáveis endógenas? Além disso, desde que as soluções possam ser consideradas relativamente precisas, o que as variáveis exógenas nestes modelos podem revelar sobre fatores que contribuíram para as principais tendências históricas? De forma geral, pode concluir-se que os modelos utilizados foram bem sucedidos na simulação das principais trajetórias macroeconômicas dos últimos cem anos. Por isso, a principal conclusão é que estas tendências são explicadas – pelo menos em parte – pelas relações macroestruturais incluídas nestes modelos.

**Abstract:**

This paper main goal is to use simple macro-econometric models built for the US and their corresponding databases to pursue a general objective in economic history. The question dealt with here is: What relevant information and conclusions may be derived from the analysis of these models and, in particular, from the dynamic patterns of their endogenous variables? Further, provided that the solutions may be considered relatively accurate, what the exogenous variables in these models can reveal about factors that contribute to determine the main historical trends? In general, it can be concluded that the models used were successful in simulating the main macroeconomic trajectories of the last hundred years. Therefore, the main conclusion is that these trends are explained – at least in part – by the macro-structural relationships included in these models.

# A hundred-year overview of the US economy: An appraisal from the perspective of simple macro-econometric models

(Preliminary Version, Oct. 2021; Revised, May 2022)

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## Abstract

This paper main goal is to use simple macro-econometric models built for the US and their corresponding databases to pursue a general objective in economic history. The question dealt with here is: What relevant information and conclusions may be derived from the analysis of these models and, in particular, from the dynamic patterns of their endogenous variables? Further, provided that the solutions may be considered relatively accurate, what the exogenous variables in these models can reveal about factors that contribute to determine the main historical trends? In general, it can be concluded that the models used were successful in simulating the main macroeconomic trajectories of the last hundred years. Therefore, the main conclusion is that these trends are explained – at least in part – by the macro-structural relationships included in these models.

## Introduction

Macro-econometric models are quantitative tools that naturally unfold from macroeconomic theories – combined with data series, estimation methods, and numerical procedures for the solution of nonlinear dynamic systems. These tools are more commonly applied to the analysis and evaluation of economic policies. From this perspective, this paper contains a not-so-common objective – that is, to use simple macro-econometric models built for the US and their corresponding databases in what is essentially an endeavor in economic history. The question pursued here is: Considering long-run trends, what relevant information and conclusions may be derived from the analysis of these models and, in particular, from the dynamic patterns of their endogenous variables? Further, provided that the solutions may be considered relatively accurate, what the exogenous variables in these systems of equations can reveal about factors that contribute to determine the main historical trends?

### Brief historical and scholarly context

From a historical perspective, macro-econometric modelling initiates in the 1940s, and from that period up to the 1960s, there was a mutually beneficial relationship between macro-econometric models and macroeconomics *per se* (De Vroey and Malgrange, 2009). The more prominent researchers in the earlier period were Jan Tinbergen, Lawrence Klein, and Arthur Goldberg.

From the 1960s to 1980s, many well-known econometrics textbooks contained thorough material on structural macro-dynamic models. Good examples are Christ (1966), Desai (1977) and Pindyck and Rubinfeld (1981). More recently, Greene (1997) includes a fairly detailed analysis of the so-called Klein Model I, which is examined in Section 1. A brief and interesting text that applies the developments in this field to macroeconomic planning is Heesterman (1970). On the other hand, a well-researched paper that examines the relevant contributions up to the 1980s is Wallis and Whitley (1991), and a more comprehensive work is Bodkin, Klein and Marwah (1991).

The emerging of macro-econometric models was accompanied by the development of a statistical framework compatible with parameter estimation in a system of interdependent equations. It is certainly not a coincidence that the Klein Model of the 1940s and the pioneering works on the simultaneous equation model for econometric estimation came into existence in the same research institute, the Cowles Foundation. This statistical framework also brought about a whole set of additional concepts – like system identification, and reduced form parameter estimation as opposed to its structural form counterpart – as well as ground-breaking estimation methods, and, to a certain degree, it dominated academic research in the field of econometrics up to the 1970s (Spanos, 1986).

From a mathematical perspective, macro-econometric models are dynamic systems of (usually) nonlinear equations whose solutions are obtained through numerical methods. These procedures can be separated into two main groups – derivative-free methods, most commonly referred to as function iteration, and procedures which use derivatives to locate the roots of a system or its fixed-points, and that are known as Newton methods (Miranda and Fackler, 2002).

### Basic macroeconomic relations from the intersectoral analysis viewpoint

In the analysis of intersectoral relations, the main variables are the levels of production in the economic sectors in a given period ($X_i$). A sector total output can be used as inputs in other sectors (variables $Z_i$), or they can meet the final sectoral demand ($Y_i$). In each sector, the equilibrium relation is total production = total demand:

$$X_i = Z_i + Y_i = Z_i + C_i + I_i + G_i + (EX–IM)_i \quad i = 1, 2, ..., n. \qquad (1)$$

On the other hand, a second equilibrium condition in each sector is total production = intermediate production + total income:

$$X_j = Z_j + W_j + \Pi_j + T_j , \qquad j = 1, 2, ..., n. \qquad (2)$$

On the right side of the system (2), in addition to the share of production used by other sectors, appear the total payments made by firms in sector $j$ in the form of salaries ($W_j$), profits ($\Pi_j$), and taxes ($T_j$). The change in the subindex for the sectors, from $i$ to j, is not casual – the equations in (1) correspond to the lines in this system, while those in (2) refer to the columns. That is, these are dual sets of equations. An interesting aspect in this development is that, if we aggregate the equations in (1) and (2), we will obtain equivalent macroeconomic relationships, showing that the national product is simultaneously equal to aggregate demand and aggregate income. It can be concluded, therefore, that the basic identities of macroeconomic analysis can be interpreted as unfolding from the principle of duality applied to a single set of intersectoral relations. This double equality is included in Klein's model of the 1940s, based on Kalecki's macroeconomic analysis,[1] which will be examined in Section 1. More specifically:

$$Y^D = C + I + G - T = \Pi + W1 + W2 \qquad (3)$$

In eq. (3), $Y^D + T = Y - (EX{-}IM)$ corresponds to the national product. In this model, on the other hand, total salaries are formed by salaries in the private sector ($W1$) and in public administration ($W2$).

## 1. Klein model based on Kalecki's macroeconomic analysis: 1920-41

The structural macro-econometric model for the American economy developed by Lawrence Klein in the second half of the 1940s is based on the macroeconomic analysis of Michal Kalecki.[2] In particular, this model presents two central characteristics of this analysis: a) inclusion of distributive elements: aggregate income is divided into the part received by entrepreneurs and individual producers (profits), the one corresponding to salaries in the private sector, and the total salaries paid by the public administration; (b) the model takes into account the capital stock, depreciation expenses and net investment – the total investment subtracted from these expenses.

The equations of the Kalecki-Klein model, classified according to their general function in a macroeconomic system, are included in Table 1. For comparison, equations in the Keynesian model, which will be examined in Section 2, are also included. The aggregate product – private national product, net of business taxes and depreciation – is equivalent to the sum of aggregate consumption, private investment, and government consumption minus direct taxes (business taxes). In each period, the value of the aggregate product is necessarily equal to the incomes generated in the production process (eq. 3 above).

Table 1 shows that the model based on Kalecki's contribution does not contain price relationships. On the other hand, the model based on Keynesian analysis contains equations to describe the average price in the economy – however, in contrast, it does not include income distribution elements, nor a variable for the stock of capital.

Endogenous variables:

$Y^D$, $C$, $I$, $\Pi$, $W1$, $K$ (capital stock at year end).

Exogenous variables:

$G$, $T$, $W2$, $t$ (year – 1939).

The series used in the model correspond to the period 1920-1941. An important aspect of this data base is that, given the data limitations in the 1940s, macroeconomic series

[1] Kalecki (1954).

[2] "Many economists will recognize the similarities between ... [this] model, the models of Kalecki's economic cycle, and some of the doctrines of Marxist economics. This model could actually be called a Marxist theory of effective demand." Klein (1950), p. 63.

were constructed with the specific purpose of corresponding to the variables of the model.[3] With the exception of $t$, all variables are in billions of 1934 dollars.

This pioneering model is linear in both the parameters and the variables. The first part of this statement indicates that the model's parameters can be estimated using methods traditionally employed in systems of interdependent equations – in particular, two-stage and three-stage least squares estimators. The second part implies that the equations can be solved using methods from linear algebra. Table 2 contains the estimation results.

To solve the Kalecki-Klein model, the equations in Table 1 must be put in matrix form. The basic matrix equation, with its solution, is presented below. These matrices, using 2SLS estimators, are included in Appendix 1.

$$\mathrm{B}y + \Gamma x = \mathbf{0}; \quad y = -\mathrm{B}^{-1}\Gamma x \tag{4}$$

Graphs 1 to 4 contain historical values for the variables $Y^D$, $I$, $P$ and $K$, and their equivalents obtained from the model solutions – in the case of $C$ and $W1$, the historical and simulated trajectories follow very closely those for $Y^D$.[4] Considering this simulated paths, it is remarkable that a model with only four exogenous variables – one of them, time represented in years – is able to reproduce cyclical movements present in the historical series with some proximity. On the other hand, it should be mentioned that three lagged variables are present in the equations, namely, $P_{-1}$, $K_{-1}$ and $(Y^D+T-W2)_{-1}$, which reveals a relatively elaborate structure of dynamic relationships, which most likely contributes to this favorable dynamic pattern.

[3] An important aspect is the "(…) interplay between the creation of the database and the model. Any macroeconometric model presupposes the existence of an economic database, but this model dates from the relatively early days of national income accounting. The data used with it had to be collected and organized as an integral part of the model building process, which the 1950 monograph describes in considerable detail." Renfro (2009, p. 1).
[4] In the Graphs, solutions based on 3SLS estimates cannot be differentiated from those based on the 2SLS method.

**Table 1**
**Equations in the Kalecki-Klein and Keynes Models**

| | Kalecki-Klein | Keynes |
|---|---|---|
| Equilibrium output and aggregate production function | $Y^D + T = C + I + G$ | $Y = C + I + G + (EX - IM)$<br>$L = f^{-1}(Y)$ |
| Effective demand and the generation of income | $C = \alpha_0 + \alpha_1 \Pi + \alpha_2 (W_1 + W_2) + \alpha_3 \Pi_{-1} + \varepsilon_C$<br>$I = \beta_0 + \beta_1 \Pi + \beta_2 \Pi_{-1} + \beta_3 K_{-1} + \varepsilon_I$<br>$W_1 = \gamma_0 + \gamma_1 (Y^D + T - W_2) + \gamma_2 (Y^D + T - W_2)_{-1} + \gamma_3 (t - 1931) + \varepsilon_W$<br>$\Pi = Y^D - W_1 - W_2$<br>$K = K_{-1} + I$ | $C = \alpha_0 + \alpha_1 (Y - T) + \varepsilon_C$<br>$I = \beta_0 + \beta_1 r + \varepsilon_I$ |
| Determination of nominal variables and price changes | | $\frac{M}{P} = \gamma_0 + \gamma_1 r + \gamma_2 Y + \varepsilon_M$<br>$w = \left[\frac{d}{dL} f\right] P$ |

**Table 2**
**Estimation results: US data, 1921-41.**

Equation for *C*

| | OLS | St. error | 2SLS | St. error | 3SLS | St. error |
|---|---|---|---|---|---|---|
| **1** | 16.237 | 1.3027 | 16.555 | 1.3208 | 16.441 | 1.3045 |
| $\Pi$ | 0.1929 | 0.0912 | 0.0173 | 0.118 | 0.1249 | 0.1081 |
| ***W1+W2*** | 0.7962 | 0.0399 | 0.8102 | 0.0402 | 0.7901 | 0.0379 |
| $\Pi_{-1}$ | 0.0899 | 0.0906 | 0.2162 | 0.1073 | 0.1631 | 0.1004 |

Equation for *I*

| | OLS | St. error | 2SLS | St. error | 3SLS | St. error |
|---|---|---|---|---|---|---|
| **1** | 10.126 | 5.4655 | 20.278 | 7.5427 | 28.178 | 6.7938 |
| $\Pi$ | 0.4796 | 0.0971 | 0.1502 | 0.1732 | -0.013 | 0.1619 |
| $\Pi_{-1}$ | 0.333 | 0.1009 | 0.6159 | 0.1628 | 0.7557 | 0.1529 |
| $K_{-1}$ | -0.112 | 0.0267 | -0.158 | 0.0361 | -0.195 | 0.0325 |

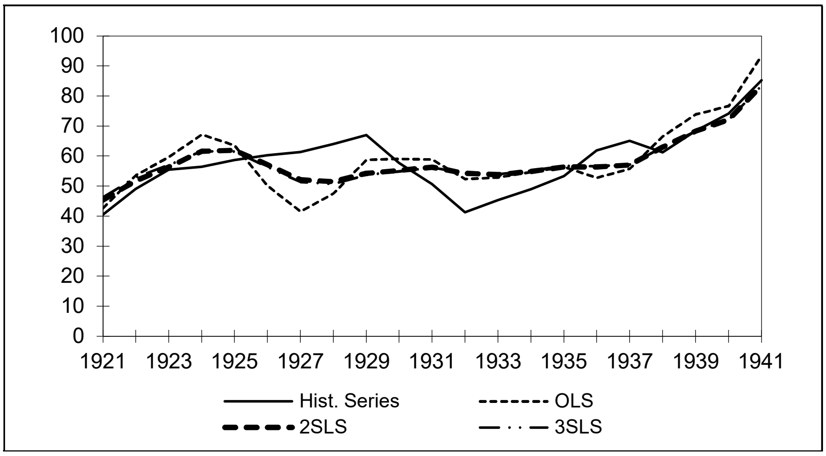
Equation for *W1*

| | OLS | St. error | 2SLS | St. error | 3SLS | St. error |
|---|---|---|---|---|---|---|
| **1** | 1.497 | 1.27 | 1.5003 | 0.0013 | 1.7972 | 1.1159 |
| $Y^D+T-W2$ | 0.4395 | 0.0324 | 0.4389 | 0.0356 | 0.4005 | 0.0318 |
| $(Y^D+T-W2)_{-1}$ | 0.1461 | 0.0374 | 0.1467 | 0.0388 | 0.1813 | 0.0342 |
| $t$**–1939** | 0.1302 | 0.0319 | 0.1304 | 0.0291 | 0.1497 | 0.0279 |

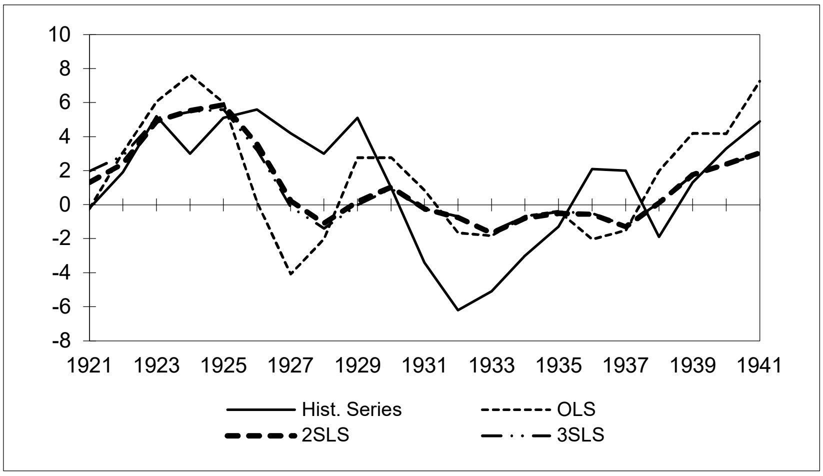
Notes: With the exception of *t*, all variables are in billions of 1934 dollars. Number of observations: 21.
Source: Estimates obtained by the author.

A characteristic of the Klein model that helps explain this favorable performance in the period examined is the dynamic property of the exogenous variable *T* – the exogenous variables are included in Graph 5 –, which presents a pattern consistent with the trajectory of the endogenous variables. Correlation coefficients between *T* and the variables $Y^D$ and $\Pi$, using first differences, are significantly different from zero.[5] And, on this point, it should be mentioned that in the worst year of the economic crisis (1932), there was a tax increase accompanied by a reduction in government consumption. This finding seems to indicate that, in addition to numerous other factors that contributed to this crisis, elements of fiscal policy also seem to have played a relevant role.

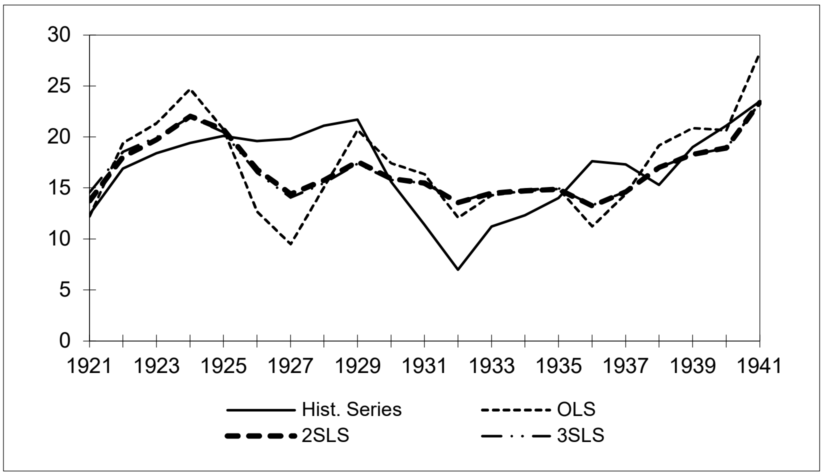
[5] The correlation coefficients, using data in first differences, are: –0,292 ($Y^D$) and –0,410 ($\Pi$). Using a one-tailed test based on simple linear regressions, the first estimate is statistically different from zero at 10% significance level, and the second at 5%.



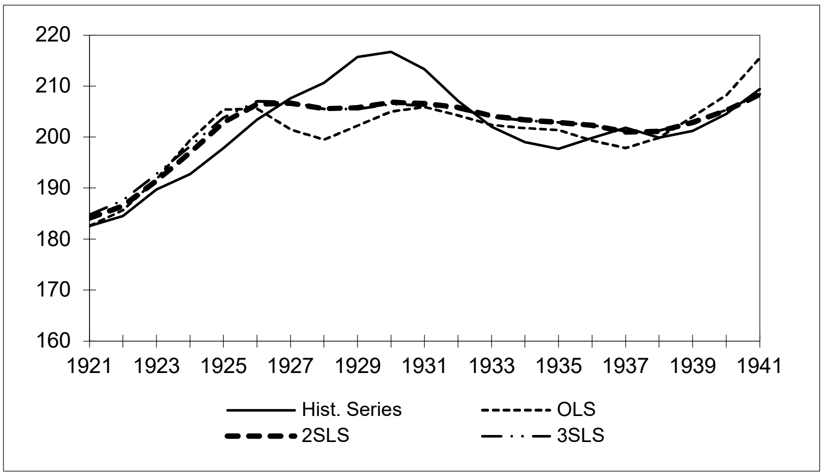

**Graph 1. Variable $Y^D$.**



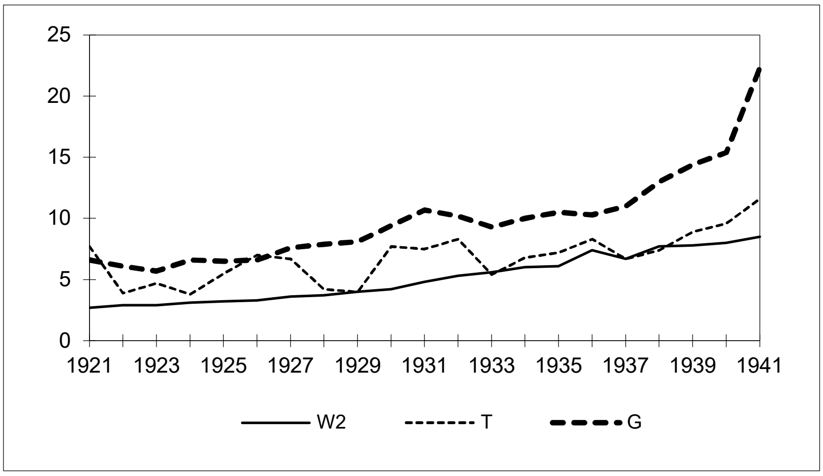

**Graph 2. Variable *I*.**




**Graph 3. Variable Π.**




**Graph 4. Variable *K*.**




**Graph 5. Exogenous variables.**

## Main conclusions from the Kalecki-Klein model and its data base

The model developed by Klein in the 1940s may be considered quite simple, since it does not include variables in current values or representing financial markets – the interest rate, for example. On the other hand, it includes the income distribution structure and the dynamics of the capital stock – both these elements are used in the model as determinants of private investment spending, net of depreciation. This model is successful, to some extent, in reproducing the historical paths of the main macroeconomic variables in the 1920s and 30s. Therefore, the main conclusion of this analysis is that non-financial elements – more directly linked to aggregate demand and income generation – contributed to the formidable economic crisis of that period.

It was argued above that the trajectory of the variable *T*, particularly in combination with government spending, has a clear pattern of pro-cyclical behavior. This is especially true in 1932, when taxation was elevated, and government spending reduced.

**2. Da Fonseca model based on Keynesian analysis: 1953-84**

The six-equation model derived from the standard Keynes' analytical system (see Table 1) was transformed into a macro-econometric model, which was estimated and solved using data available for the US economy, for the 1953-84 period (da Fonseca, 2017a). In this development, two main changes were introduced. First, the equation for $L$ (total employment) was used in the equation for $w$ (average nominal wage). That is, the variable $L$ was eliminated:

$$L = f^{-1}(Y)\,; \qquad w = \left[\frac{d}{dL} f\right] P \quad \rightarrow \quad w = g(Y)P \tag{5}$$

The second change is the introduction of $Y$ (aggregate product) in the equation for $I$ (private investment), both in the current period and with a one-period lag – see eq. (8) below. The rationality for this specification is that $\beta_0 + \beta_3 Y_{-1}$ is a proxy for $I_{-1}$. Hence the left side of (8) contains approximately $\Delta I$ – which is modeled through $r$ and $\Delta Y$.

Equations of the macro-econometric model:

$$Y = C + I + G + (EX - IM) \tag{6}$$

$$C = \alpha_0 + \alpha_1 (Y - T) + \varepsilon_C \tag{7}$$

$$I = \beta_0 + \beta_1 r + \beta_2 \Delta Y + \beta_3 Y_{-1} + \varepsilon_I \tag{8}$$

$$\frac{M}{P} = \gamma_0 + \gamma_1 r + \gamma_2 Y + \varepsilon_M \quad \rightarrow \quad r = \left(\frac{M}{P} - \gamma_0 - \gamma_2 Y - \varepsilon_M\right)\frac{1}{\gamma_1} \tag{9}$$

$$\ln\left(\frac{w}{P}\right) = \zeta_0 + \zeta_1 \ln Y + \varepsilon_w \tag{10}$$

A description of the variables in the model, with their units of measurement, follows.

Endogenous variables:

$Y$: Aggregate income and product, in real values;
$C$: Private consumption, in real values;
$I$: Aggregate investment, in real values;
$r$: Interest rate, in real terms;
$P$: Average price level.

Exogenous variables:

$G$: Government spending, in real values;
$EX\text{–}IM$: Trade balance in national currency, real values;
$T$: Taxes, real values;
$M$: Money stock, current values;
$w$: Average wage rate, current values.

To a large extent, the contents of any standard macroeconomics textbook are related to the concepts and analyses arising from these five equations. In especial, the combination of equations (6) to (9) produces the standard IS-LM analysis of output and interest rate equilibrium. On the other hand, eq. (10) is usually dealt with independently of the rest of the system and, therefore, the price level is treated in textbook analyses as an exogenous element.

Eq. (8) includes an endogenous variable with a one period lag [6] and, therefore, this set of equations is a dynamic system – from a mathematical standpoint, a system of nonlinear difference equations, and from an economic point of view, it is a structural nonlinear dynamic macro-econometric model.

### Data set and parameter estimation

Usually, one of the most difficult and challenging tasks in developing a macro-econometric model is to assemble the data base for its variables. However, in the case of the model used here, this was one of the easiest parts. The series drawn from data available in a couple of traditional textbooks – appropriately, one of them on (macro) economics and the other on econometrics.[7] These series appear in Appendix 2.

The parameters were estimated using data for the 1953-79 period – the last five years available in each series were not used. The reason is that this part of the data was reserved so that it could be applied to the evaluation of the model as a prediction tool – that is, beyond the period used in the estimation process. The estimates appear in Table 3. Given that the 2SLS method provides parameter estimates with the correct first-derivative signs, they were used in the solution of the model, which is described below.

**Table 3. Estimation results. US data, 1954-79.**

| . | | 2SLS estimator | | 3SLS estimator | |
|---|---|---|---|---|---|
| Regression eq. | | Coef. | St. error | Coef. | St. error |
| ***C*** | 1 | 8.052 | 5.297 | 8.910 | 5.484 |
| | $Y–T$ | 0.8947 | 0.0076 | 0.8934 | 0.0079 |
| ***I*** | 1 | –6.437 | 12.591 | –5.876 | 11.527 |
| | $r$ | –3.394 | 4.2453 | –3.443 | 3.7754 |
| | $\Delta Y$ | 0.5975 | 0.1182 | 0.7136 | 0.1051 |
| | $Y_{-1}$ | 0.1503 | 0.0095 | 0.1459 | 0.0089 |
| ***r*** | 1 | 1.162 | 0.9955 | 2.474 | 1.448 |
| | $M/P$ | –0.0173 | 0.00978 | 0.0051 | 0.0108 |
| | $Y$ | 0.00423 | 0.00277 | –0.0017 | 0.0033 |
| **ln(*w*/*P*)** | 1 | 1.832 | 0.1551 | 1.823 | 0.2323 |
| | $\ln Y$ | 0.3870 | 0.0226 | 0.3883 | 0.0338 |

*Source*: Estimates obtained by the author.

### Solution of a system of nonlinear equations [8]

Solution methods of nonlinear systems of equations are based upon – with a decreasing level of generality – fixed-point theorems, the method of successive approximations,

[6] Eq. (8), which shapes the dynamics of the model, plays a unique role in this system – it is actually an example of the "art" of specifying econometric equations. If the model's dynamics works appropriately, it should reproduce with some success the time path of the real macroeconomic variables.
[7] The references are W. J. Baumol and A. S. Blinder, *Economics – Principles and Policy*, 3.ª ed., and Greene (1997). In the first reference, there are data for *Y*, *C*, *I*, *G*, (*EX–IM*), nominal *r*, *w* and *P*. In the second, there are data for (*Y–T*) and *M* /*P*.
[8] Each simulation period contains a problem to be solved.

and so-called Newton methods.[9] Fixed-point theorems establish that, under certain conditions, the solution of the equation $x = f(x)$ exists. One important aspect of this analysis is that any equation can be put in the form of the fixed-point problem. For example, if the original equation is $g(x) = 0$, then a corresponding fixed-point equation is:

$$x = x + g(x) \tag{11}$$

The method of successive approximations is, at the same time, one way to prove a fixed-point theorem and a method to find a solution. In this iterative scheme, one computes the successive approximations:[10]

$$x_{n+1} = f(x_n); \quad n = 0, 1, 2, \ldots \tag{12}$$

The general method associated to Newton is an important example of the iterations in (12). Representing the $i$–th equation in a nonlinear system by $g_i(\boldsymbol{x}) = 0$, where the symbol $\boldsymbol{x}$ represents the set (vector) of endogenous variables, the method provides the solution in each stage – or approximation – ($\boldsymbol{x}^{k+1}$) from the previous one ($\boldsymbol{x}^k$) through the following linear system:

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - (J^k)^{-1}\, \boldsymbol{g}^k \tag{13}$$

In eq. (13), $J^k$ represents the Jacobian matrix (of first derivatives) obtained in stage $k$, and $\boldsymbol{g}^k$ is a vector with the results calculated at the same stage for the functions $g_i$. Usually, this procedure based on derivatives converges quickly for the correct solution. One difficulty with this method is that the Jacobian matrix must be calculated at each step and, naturally, in a more convenient version of the method – albeit one in which convergence to the solution is not so quick, neither so much guaranteed – this matrix is fixed. In the so-called *modified* Newton method, only the Jacobian matrix for the initial stage (0) is used:

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - (J^0)^{-1}\, \boldsymbol{g}^k \tag{14}$$

The modified Newton method is quite easily translated into computer code – in the case of the model solved here, a spreadsheet was used to provide the solutions. This arrangement is very convenient, since the construction of data series for the variables, the estimation of the equations' parameters, and the model's dynamic solution can all be performed in different spreadsheets of the same file.

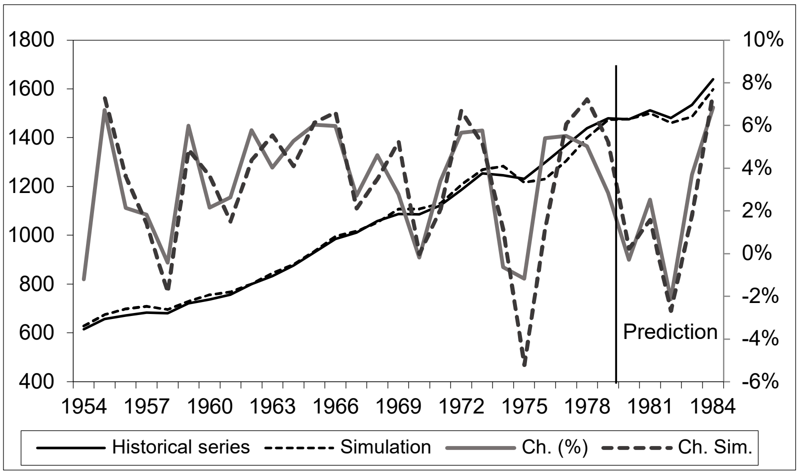
The first period for which the model was solved was 1954, using data for the previous year – which also provided a starting solution for the successive-approximations method. After that, the solution obtained in one period was used as a starting point for the following year. In each period, the modified Newton method was utilized and, as a rule, only a few iterations were necessary to achieve convergence. Therefore, in terms of data for the endogenous variables, only information for 1953 was, in fact, used. In

[9] Franklin (1980), chap. 3, and Strang (1986), chap. 5.

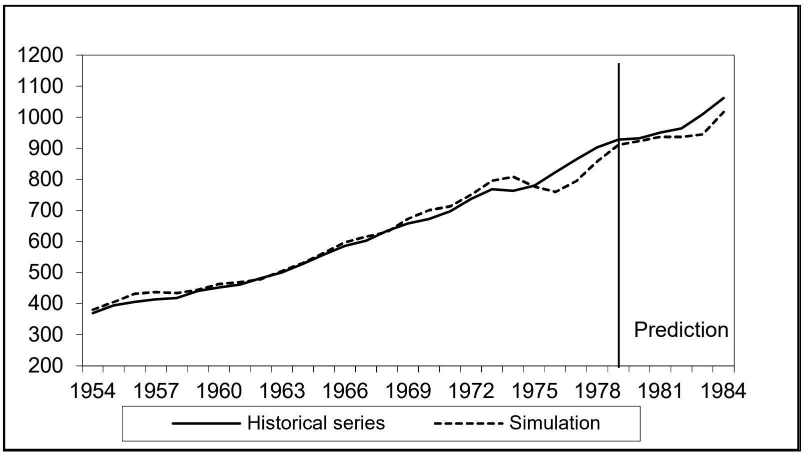
[10] To prove the fixed-point theorem, it is sufficient to establish the convergence of the sequence $x_n$. One point that should be made is that eq. (12) can be applied directly to a nonlinear problem – that is, without using derivatives. This approach to find a solution, which is also known as function iteration, is very straightforward, although it does not work so often in more complex nonlinear problems.

other words, the solution values obtained from the model reflect only the dynamic properties of the nonlinear system and the trends of the exogenous variables – which are quite limited in number (only five).

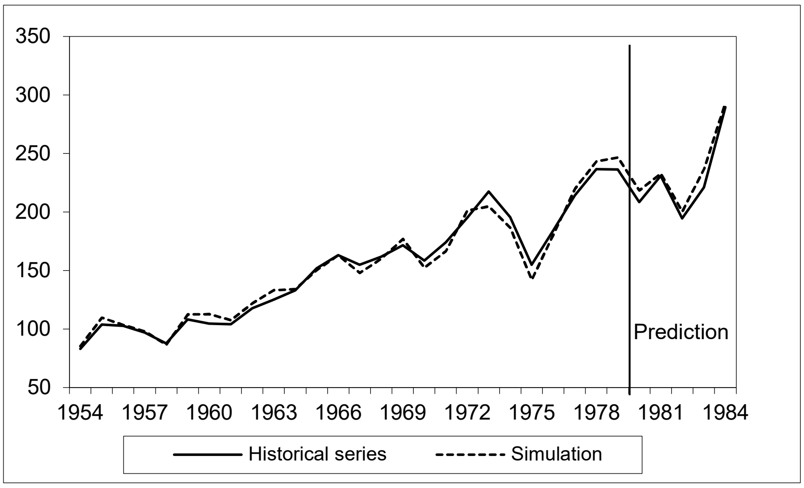
The solutions obtained from this five-equation macro-econometric model, together with the historical series, are displayed in the Graphs below. For a description of the units used in the data, see Appendix 2. The "prediction" period contains "out of sample" data – *i. e.*, data that were not used in parameter estimation.



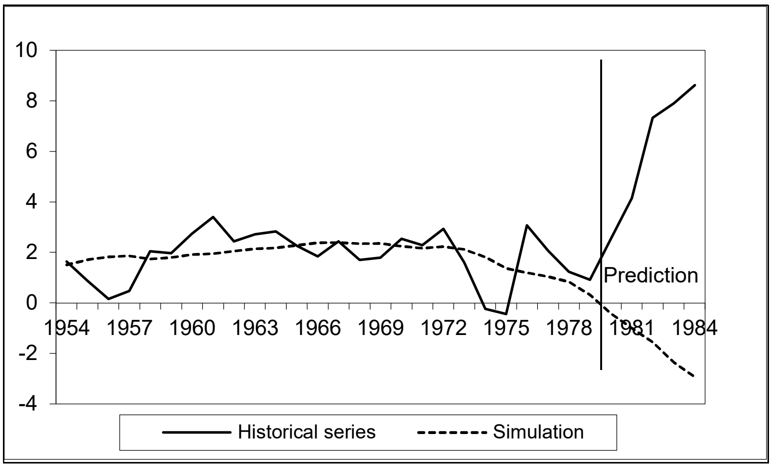

**Graph 6. Variable *Y*.**



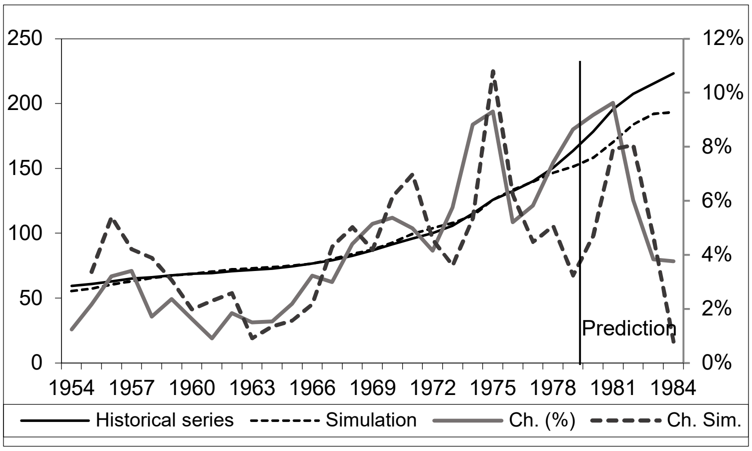

**Graph 7. Variable *C*.**




**Graph 8. Variable *I*.**




**Graph 9. Variable *r*.**



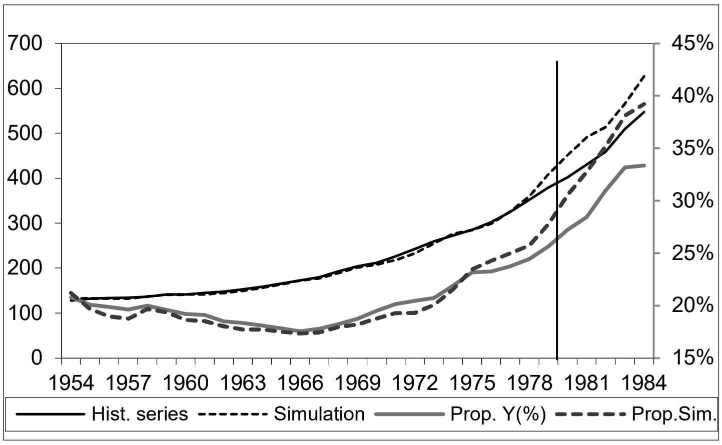

**Graph 10. Variable *P*.**

### Main conclusions from the Keynes-da Fonseca model and its data base

Considering the Graphs in this Section, the most significant is probably the one for aggregate investment ($I$) – the true dynamic variable in the model. Based on Graph 8, one can conclude that the model captured fairly well the dynamic path of the US economy in the three decades since the middle of the 1950s. Overall, the same conclusion applies to the other endogenous variables, with the significant exception of the real exchange rate ($r$). Nevertheless, one should not over emphasize this aspect of the model's solutions – that they were quite close to historical values –, given the especial nature of the exogenous variables. Although these variables are limited in number, each one represents fundamental and complex parts of the macroeconomic system.

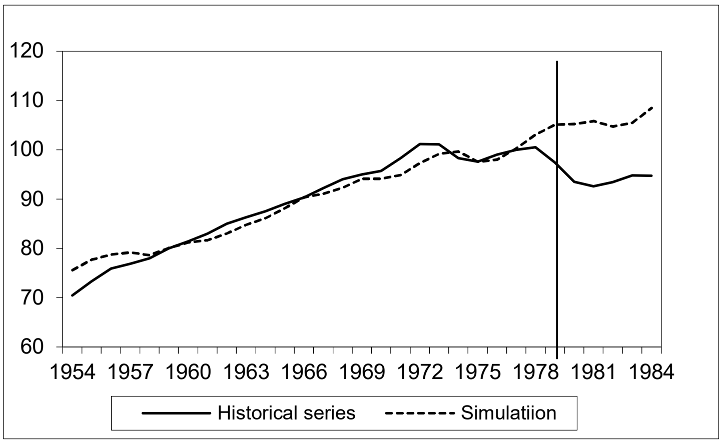
Probably, the most valuable feature of a structural macro-econometric model is that it reveals, among hundreds of different types of information available to researchers, what we should be concentrating on. More explicitly, if the model reproduces reasonably well the pattern of real macroeconomic data, and given that the model's solutions derive from the exogenous variables and the model's dynamic properties, then it is a logical conclusion that these exogenous variables are the really important ones – and there are only five of them to keep track of. In other words, one of the main accomplishments of this analysis is that it reveals to researchers and professional analysts that they should pay especial attention to the trends of average nominal wage, trade balance, government spending, taxes, and money supply.

One additional aspect is that the dynamic paths of money supply and nominal wage rate are truly consistent with Keynesian theory – which indicates that economic growth is usually accompanied by increases in money supply and a stable or declining real wage rate.[11] These patterns suggested by theory are duly observed in the period examined (see Graphs 12 and 13).




**Graph 11. Variable *M*/*P*.**

[11] In the first decade and a half of the period analyzed, there was, in fact, an increase of the average real wage.



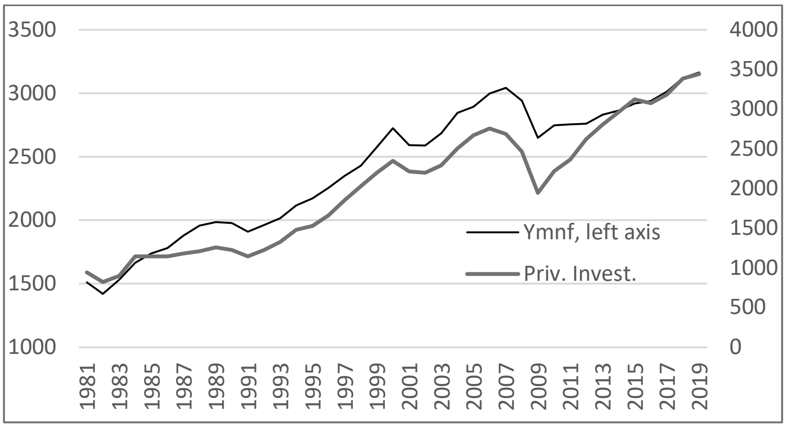

**Graph 12. Variable *w/P*.**

## 3. A model containing income distribution, price changes and Government financing: 1980-2019

In Klein's model based on Kalecki, two identities are included for the aggregate product – it is equal to both aggregate demand and total income. On the other hand, in the model of Section 2, only the first of these equalities is present. The model of this Section, in turn, includes an equilibrium relationship between aggregate production and the sum of the outputs in the main economic sectors.[12] This specification, following the logic of intersectoral analysis, allows to establish the connection between the aggregate product of the sectors and the level of employment (eq. 2 in Appendix 3). Moreover, the equality between production and aggregate income, separated into profits and salaries, is also included implicitly – this relationship is used in eq. 8. This set up draws from the intersectoral analysis developed originally by Wassily Leontief and, in special, the version in which production, demand and income distribution are treated as an ensemble of intersectoral relations (Myiazawa, 1976).

The sectoral outputs are determined directly by income levels generated in the production process – that is, the consumption function is not included explicitly. Moreover, the lack of a function for private investment – which is also internalized in the system – will probably astonish quite a few people. However, such criticism is not exactly appropriate in this case. The reason is that one of the variables included, i.e., production in the manufacturing sector ($Y_{MNF}$), has a dynamic trajectory almost identical to private investment. Therefore, to a large extent, including that variable means to include investment. This relationship is illustrated in Graph 13 (the correlation coefficient in the period described is 0.966).

[12] The complete set of equations of this model are included in Appendix 3.




**Graph 13. Data series, $Y_{MNF}$ and private investment.**

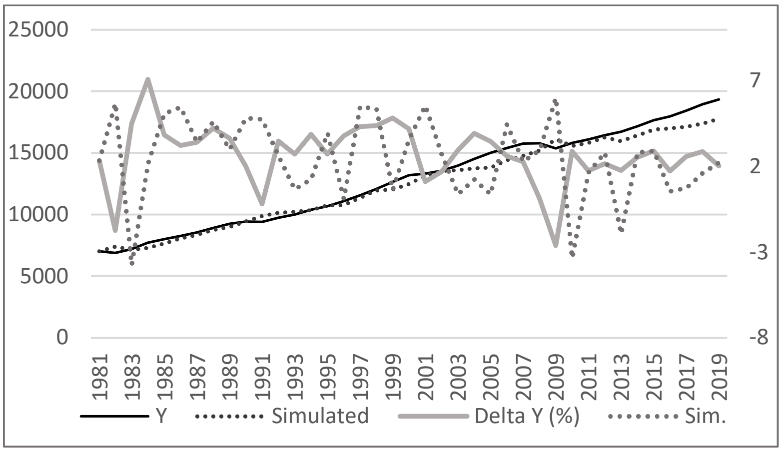
In this macro-econometric model, nominal values and price changes are determined by equations representing, simultaneously, the fiscal balance of the government, the money supply, and the trajectory of a price index. In the latter, both the monetary component of price changes and the cost factors – in particular, the average wage – are introduced for describing the dynamic path of the general price level.[13]

### Analysis of the model's solutions and its database

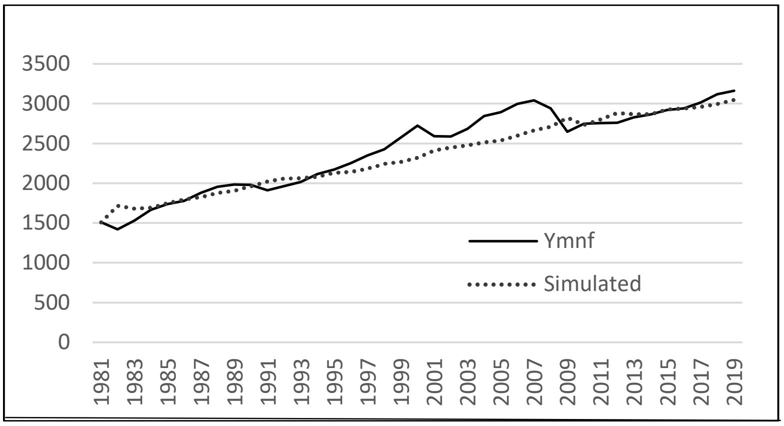
The data base for this macro-econometric model was built from series available in the data bank of the St. Louis Fed (https://fred.stlouisfed.org), and also from data available on the website of the Bureau of Economic Analysis (www.bea.gov). Solutions were obtained both through function iteration (see Note 10) and the Jacobian matrices of the system (Newton method, see Section 2). These matrices were constructed for a few given periods, separated by intervals of approximately ten years. Nevertheless, the method of function iteration, which is much simpler to use, since it does not require derivatives, was the preferred alternative. The solutions represented in Graphs 14 to 22 were obtained from this solution method.

The current version of the model's data bank revealed some deficiencies which led to a change in the system of equations described in Appendix 3 – specifically, the removal of eq. 11, since its inclusion with the data set presently available caused the virtual failure of the solution procedures.[14] Graphs 14 to 22 describe the main variables and the corresponding solutions. The initial solution period is 1982 and, therefore, real data were used for the endogenous variables only in 1980-81 – for other periods, the initial solution was provided by the previous year's solution.

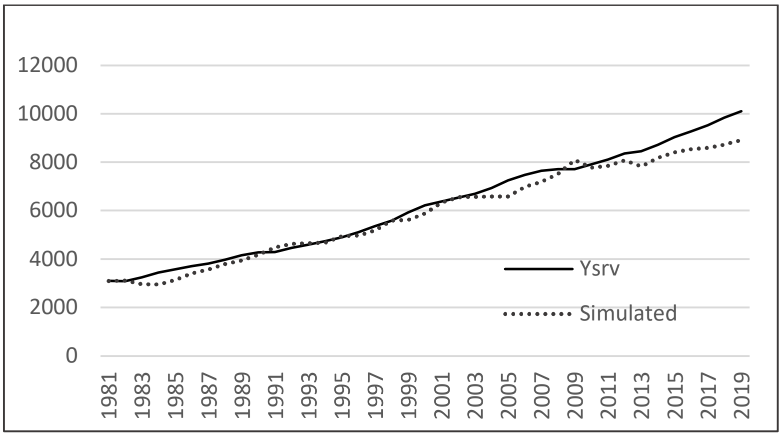
[13] Equations 14 and 15 in Appendix 3, which simulate the dynamic paths of the average price level ($P$) and the average wage ($w$), are based on the econometric analysis in da Fonseca (2017b).
[14] That is, the corresponding variable had to be treated as exogenous. For further details on the model's equations and variables, its data base and estimation methods, as well as the procedures for solution, see da Fonseca (2021).




**Graph 14. Variable *Y*.**

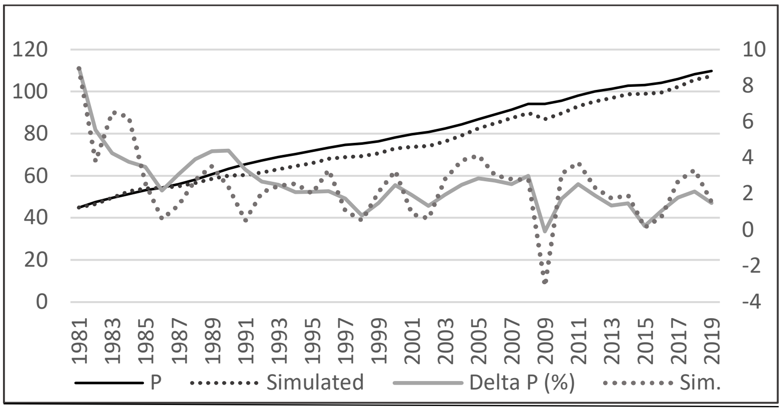



**Graph 15. Variable $Y_{MNF}$.**

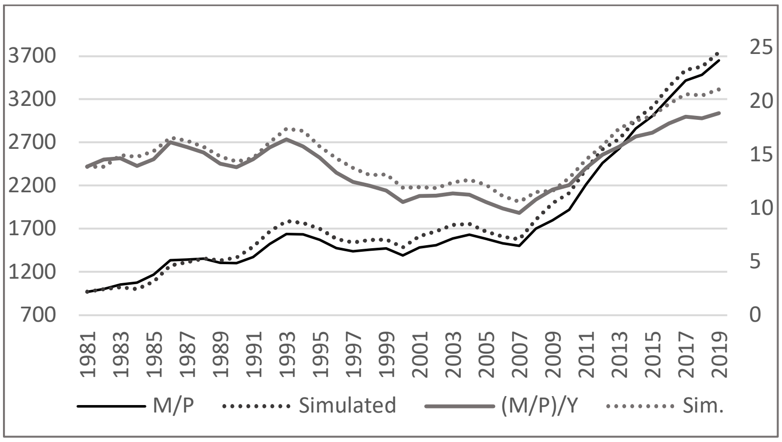



**Graph 16. Variable $Y_{SRV}$.**

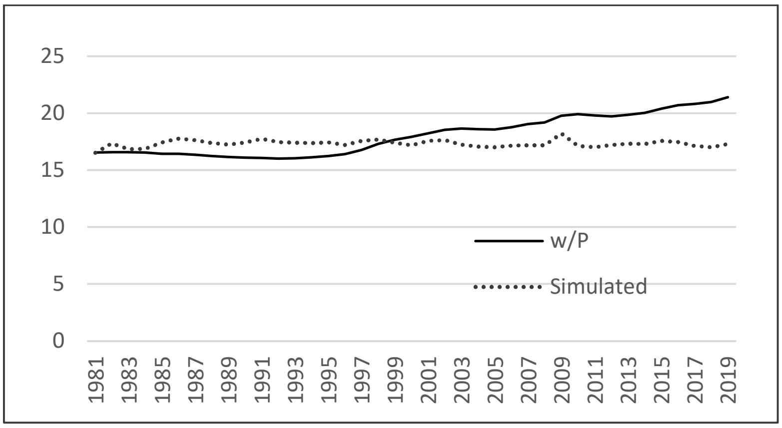

**Graph 17. Variable *r*.**



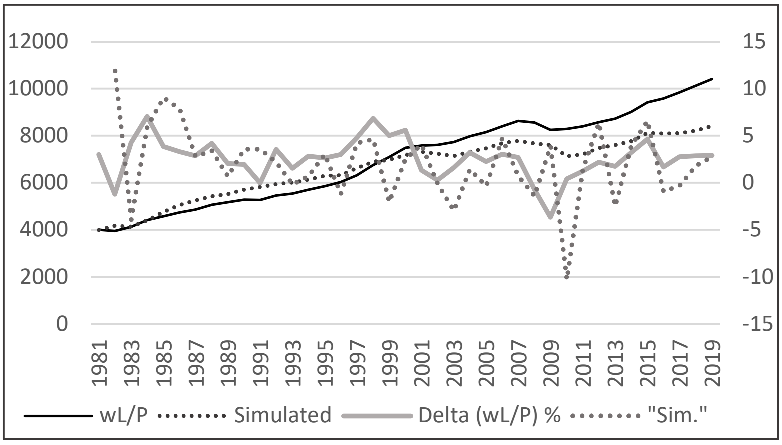

**Graph 18. Variable *P*.**



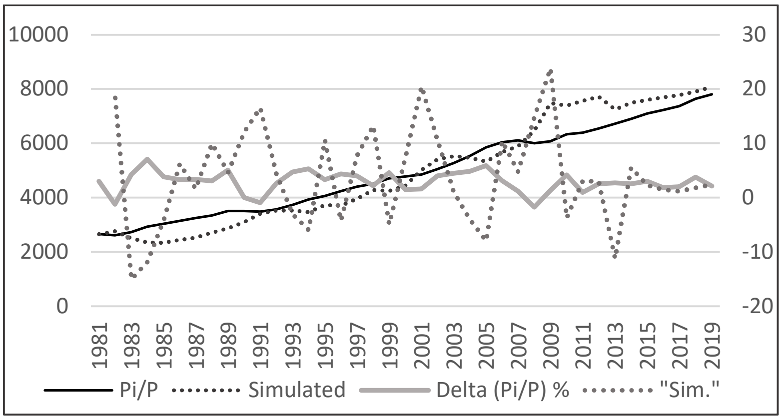

**Graph 19. Variable *M/P*.**




**Graph 20. Variable *w/P*.**




**Graph 21. Variable *wL /P*.**

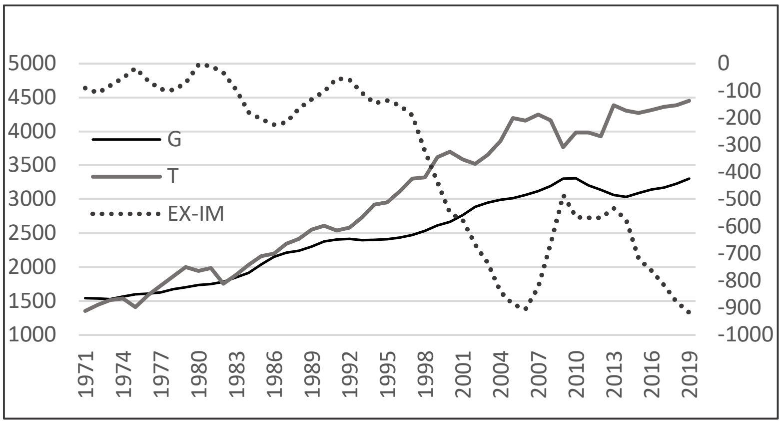



**Graph 22. Variable Π */P*.**

**Main conclusions from the model in this Section and its data base**

Significant recessions occurred during the period for which the model was solved: in 1981-82, 1990-91, 2001, and 2008-9. As a rule, the model was not successful in simulating the change of economic activity in those years. In the case of the recessions that had a stronger financial component, this shortcoming might be expected – in fact, it would be quite unlikely that a structural model, centered on outputs of sectors and income distribution, could simulate the financial crisis of 2008-9. In relation to this point, one important exception is the simulation of variable *P*. To a certain extent, the model was indeed successful in describing the trajectory of the average price level, even in the recessions of 2001 and 2008-9 (with some overshooting), although the same cannot be said for the average wage (*w*) – see Graphs 18 and 20.

In any case, one aspect that should be emphasized is that the general macroeconomic tendencies of the last four decades were captured by this model, which indicates that, in fact, its exogenous variables contributed to the overall macroeconomic trajectories. These variables are listed in Appendix 3, and three of them are displayed in Graph 23, namely *G*, *T* and *EX–IM*. This Graph illustrates a regular and relatively strong increase of taxation, except in the years of recessions. A more moderate, and relatively stable, growth of Government spending is also observed, except in the period 2011-14. On the other hand, Graph 23 shows a very irregular path of net exports, with a sharp decline from 1997-2006, a significant upsurge in the next three years, and a renewed downward movement since 2014.




**Graph 23. Exogenous variables, 1971-2019: *G*, *T* and *EX–IM*.**

## 4. Concluding remarks

The analysis in this paper indicates that quite different structural models – even though with similar theoretical relations, centered on the contributions of Kalecki, Keynes and Leontief – were successful in simulating the main macroeconomic trends in the US over a hundred-year period.

The model examined in Section 1, elaborated by Klein in the 1940s, and which does not include financial variables, was able to reproduce to a certain degree the very strong movements in aggregate income and investment observed in the 1920s and 30s. The conclusion that may be drawn is that part of these movements can be explained by typical macroeconomic factors, linked to aggregate demand, income generation and distribution, and the path of capital stock.

The model examined in Section 2, in turn, based on Keynesian analysis, reproduces in a surprisingly accurate fashion the trajectory of investment from the 1950s to 1980s. The simulation of the dynamic path of the average price is not so perfect, but it is still appropriate to conclude that Keynesian analysis can successfully explain and simulate the combination of reduced growth and high inflation that prevailed in the 1970s.

Finally, the model used to simulate macroeconomic trends since the 1980s seeks to combine the main characteristics of the two previous models. On the other hand, it departs from them by including elements of intersectoral analysis, equations to represent government financing, and both monetary and cost factors in the determination of the average price level. This model is not so successful in reproducing the cyclical movements of the U.S. economy, but in the case of the trajectory of the average price level, its performance can be considered relatively accurate.

## Appendix 1

## Matrices in the Kalecki-Klein model – 2SLS estimators

Beta / Gamma

| | *C* | *I* | *W1* | *Y* | *P* | *K* | | 1 | *W2* | *T* | *G* | *t* | $P_{-1}$ | $K_{-1}$ | $(Y^D+T-W2)_{-1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *C* | -1 | 0 | 0,8102 | 0 | 0,0173 | 0 | | 16,555 | 0,8102 | 0 | 0 | 0 | 0,2162 | 0 | 0 |
| *I* | 0 | -1 | 0 | 0 | 0,1502 | 0 | | 20,278 | 0 | 0 | 0 | 0 | 0,6159 | -0,158 | 0 |
| *W1* | 0 | 0 | -1 | 0,4389 | 0 | 0 | | 1,5003 | -0,439 | 0,4389 | 0 | 0,1304 | 0 | 0 | 0,1467 |
| *Y* | 1 | 1 | 0 | -1 | 0 | 0 | | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 |
| *P* | 0 | 0 | -1 | 1 | -1 | 0 | | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 |
| *K* | 0 | 1 | 0 | 0 | 0 | -1 | | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

$Beta^{-1}$ / $-Beta^{-1} \times Gamma$

| | *C* | *I* | *W1* | *Y* | *P* | *K* | | 1 | *W2* | *T* | *G* | *t* | $P_{-1}$ | $K_{-1}$ | $(Y^D+T-W2)_{-1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *C* | -1,664 | -0,664 | -1,219 | -0,664 | -0,128 | 0 | | 42,83 | 0,6842 | -0,128 | 0,6637 | 0,159 | 0,7685 | -0,105 | 0,1789 |
| *I* | -0,153 | -1,153 | 0,0518 | -0,153 | -0,176 | 0 | | 25,84 | -0,029 | -0,176 | 0,1531 | -0,007 | 0,7433 | -0,182 | -0,008 |
| *W1* | -0,797 | -0,797 | -1,512 | -0,797 | -0,134 | 0 | | 31,64 | -0,151 | -0,134 | 0,7974 | 0,1972 | 0,6635 | -0,126 | 0,2219 |
| *Y* | -1,817 | -1,817 | -1,168 | -1,817 | -0,304 | 0 | | 68,67 | 0,6552 | -1,304 | 1,8168 | 0,1523 | 1,5118 | -0,287 | 0,1713 |
| *P* | -1,019 | -1,019 | 0,3448 | -1,019 | -1,171 | 0 | | 37,03 | -0,193 | -1,171 | 1,0194 | -0,045 | 0,8482 | -0,161 | -0,051 |
| *K* | -0,153 | -1,153 | 0,0518 | -0,153 | -0,176 | -1 | | 25,84 | -0,029 | -0,176 | 0,1531 | -0,007 | 0,7433 | 0,818 | -0,008 |

## Appendix 2

## Data for major macroeconomic variables: USA, 1953-84[1]

| | *Y* | *C* | *I* | *G* | *(EX-IM)* | *(Y-T)* | $r_{nom}$ [2] | *P* [3] | *M/P* [4] | *w* [5] |
|---|---|---|---|---|---|---|---|---|---|---|
| 1953 | 623.6 | 363.4 | 85.3 | 170.1 | 4.8 | 399.1 | 1.62 | 58.82 | 126 | 39.93 |
| 1954 | 616.1 | 370 | 83.1 | 156 | 6.9 | 403.6 | 1.64 | 59.55 | 128 | 41.48 |
| 1955 | 657.5 | 394.1 | 103.8 | 152.3 | 7.3 | 427 | 0.87 | 60.84 | 132 | 44.07 |
| 1956 | 671.6 | 405.4 | 102.6 | 153.5 | 10.1 | 446.5 | 0.15 | 62.79 | 133.5 | 47.09 |
| 1957 | 683.8 | 413.8 | 97 | 161.2 | 11.8 | 455.2 | 0.47 | 64.93 | 134.1 | 49.34 |
| 1958 | 680.9 | 418 | 87.5 | 169.8 | 5.6 | 461 | 2.05 | 66.04 | 136 | 50.9 |
| 1959 | 721.7 | 440.4 | 108 | 170.6 | 2.7 | 479.3 | 1.97 | 67.6 | 141.4 | 53.44 |
| 1960 | 737.2 | 452 | 104.7 | 172.8 | 7.7 | 489.6 | 2.74 | 68.7 | 141.4 | 55.26 |
| 1961 | 756.6 | 461.4 | 103.9 | 182.9 | 8.5 | 503.9 | 3.4 | 69.33 | 144.5 | 56.86 |
| 1962 | 800.3 | 482 | 117.6 | 193.2 | 7.5 | 524.8 | 2.44 | 70.61 | 148 | 59.31 |
| 1963 | 832.5 | 500.5 | 125.1 | 197.6 | 9.4 | 542.7 | 2.72 | 71.67 | 152.6 | 61.12 |
| 1964 | 876.4 | 528 | 133 | 202.6 | 12.8 | 580.5 | 2.82 | 72.77 | 158.6 | 62.92 |
| 1965 | 929.3 | 557.5 | 151.9 | 209.8 | 10.1 | 616.3 | 2.26 | 74.36 | 165.5 | 65.4 |
| 1966 | 984.8 | 585.7 | 163 | 229.7 | 6.5 | 647 | 1.84 | 76.76 | 172.8 | 68.49 |
| 1967 | 1011.4 | 602.7 | 154.9 | 248.5 | 5.4 | 673.1 | 2.44 | 79.06 | 180 | 72.03 |
| 1968 | 1058.1 | 634.4 | 161.6 | 260.2 | 1.9 | 701.4 | 1.7 | 82.54 | 192.7 | 76.67 |
| 1969 | 1087.6 | 657.9 | 171.4 | 257.4 | 0.9 | 722.7 | 1.79 | 86.79 | 203.8 | 81.47 |
| 1970 | 1085.6 | 672.1 | 158.5 | 251.1 | 3.9 | 751.7 | 2.53 | 91.45 | 211.6 | 86.48 |
| 1971 | 1122.4 | 696.8 | 173.9 | 250.1 | 1.6 | 779.1 | 2.29 | 96.01 | 226.2 | 93.26 |
| 1972 | 1185.9 | 737.1 | 195 | 253.1 | 0.7 | 810.3 | 2.93 | 100 | 242.6 | 100 |
| 1973 | 1254.3 | 767.9 | 217.5 | 253.3 | 15.5 | 865.2 | 1.6 | 105.75 | 259.7 | 105.65 |
| 1974 | 1246.3 | 762.8 | 195.5 | 260.3 | 27.8 | 857.7 | -0.23 | 115.08 | 272.6 | 111.78 |
| 1975 | 1231.6 | 779.4 | 154.8 | 265.2 | 32.2 | 874.8 | -0.44 | 125.79 | 285.4 | 121.32 |
| 1976 | 1298.2 | 823.1 | 184.5 | 265.2 | 25.4 | 906.9 | 3.06 | 132.34 | 301.9 | 129.46 |
| 1977 | 1369.7 | 864.3 | 214.2 | 269.2 | 22 | 943.3 | 2.07 | 140.05 | 325.2 | 138.39 |
| 1978 | 1438.6 | 903.2 | 236.7 | 274.6 | 24 | 988.6 | 1.23 | 150.42 | 351.7 | 149.38 |
| 1979 | 1479.4 | 927.6 | 236.3 | 278.3 | 37.2 | 1015.5 | 0.91 | 163.42 | 379 | 157.28 |
| 1980 | 1475 | 931.8 | 208.5 | 284.3 | 50.3 | 1021.7 | 2.53 | 178.42 | 401.5 | 164.84 |
| 1981 | 1512.2 | 950.5 | 230.8 | 287 | 43.8 | 1049.7 | 4.14 | 195.6 | 430.1 | 178.98 |
| 1982 | 1480 | 963.3 | 194.4 | 292.7 | 29.7 | 1058.5 | 7.33 | 207.38 | 458.5 | 191.4 |
| 1983 | 1534.7 | 1009.2 | 221.1 | 291.9 | 12.6 | 1095.5 | 7.9 | 215.34 | 509.2 | 201.72 |
| 1984 | 1639.9 | 1062.4 | 289.6 | 302.1 | -14.2 | 1169.1 | 8.62 | 223.44 | 547.3 | 209.09 |

1. Variables in billions of 1972 dollars, $r_{nom}$ in %, *P* and *w* are indices (1972=100).
2. Average interest rates on bonds issued by top level companies.
3. GNP deflator.
4. $M_1$ stock.
5. Average hourly nominal wage.

Source: W. J. Baumol and A. S. Blinder, *Economics – Principles and Policy*, 3rd ed. (N. York, Harcourt Brace Jovanovich, 1985), and Greene (1997).

## Appendix 3

## Equations of the macro-econometric model with income distribution, price changes and Government financing

| | |
|---|---|
| Equilibrium output and aggregate production function | 1. $Y = Y_{AGR} + Y_{MNF} + Y_{TRD} + Y_{SRV} + Govt\ Sector$<br>2. $L = a_{w1}\ Y_{AGR} + a_{w2}\ Y_{MNF} + a_{w3}\ Y_{TRD} + a_{w4}\ Y_{SRV}$ |
| Effective demand and the generation of income | 3. $Y_{MNF} = b_0 + b_1\ Y_{-1} + b_2\ \Delta\ (wL/P - Tw) + b_3\ \Delta\ (EX–IM) + \varepsilon_3$<br>4. $Y_{TRD} = b_0 + b_1\ Y_{-1} + b_2\ \Delta\ (wL/P - Tw) + \varepsilon$<br>5. $Y_{SRV} = b_0 + b_1\ Y_{-1} + b_2\ \Delta\ (wL/P - Tw) + b_3\ \Delta\ (\Pi/P - T_{\Pi}) + b_4\ r + \varepsilon$<br>6. $r = b_0 + b_1\ M/P + b_2\ Y + \varepsilon$<br>7. $wL/P = (w\ L\ Kw) / P$<br>8. $\Pi/P = Y - IndTax - wL/P$ |
| Determination of nominal variables and price changes | 9. $M = \mu\ B$<br>10. $B = \alpha\ (B + Debt)$<br>11. $B + Debt = (B + Debt)_{-1} + G\ P + Interest + Subsids - T\ P$<br>12. $Interest = \left[\left(\frac{P}{P_{-1}}\right)\left(1 + \frac{r}{100}\right) - 1\right] Debt_{-1}$<br>13. $Debt = (1 - \alpha)\ (B + Debt)$<br>14. $\ln P = \ln P_{-1} + b_1\ [\Delta \ln (M\ V) - \Delta \ln Y] + b_2\ \Delta \ln w + b_3\ \Delta \ln Inputs + b_4\ \Delta \ln e + \varepsilon$<br>15. $\ln w = \ln w_{-1} + b_1\ \Delta \ln P_{-1} + \varepsilon$ |

Note: All variables are in billions of 2012 dollars, except if stated otherwise.

Endogenous variables:

- *Y* Gross National Product;
- *L* Total employment (thousands of individuals);
- $Y_{MNF}$ Real value added in manufacture, utilities and construction;
- $Y_{TRD}$ Real value added in trade and transportation;
- $Y_{SRV}$ Real value added in services;
- *r* Interest rate on corporate bonds (Moody's Aaa rating), deflated by GDP price index (%);
- *wL/P* Total compensation of employees;
- Π/*P* Total gross operating surplus;
- *M* M1 (billions of dollars in December);
- *B* Monetary base (billions of dollars in December);
- *Interest* Federal Government interest payments (billions of dollars in December);
- *Debt* Total federal debt (billions of dollars in December);
- *P* GDP price index (2012 = 100);
- *w* Average earnings of non-supervisory employees (dollars per hour).

Exogenous variables and varying parameters:

- $Y_{AGR}$ Real value added in agriculture, forestry, fishing, and hunting;
- *G* Government purchases;
- *EX-IM* Net exports;
- *T* Real taxes;
- *IndTax* Tax on products less subsidies
- *Subsids* Federal Government subsidies (billions of dollars);
- *Govt sector* Real value added by the Government sector;
- *Inputs* All commodities, Producer Price Index;
- *e* Trade weighted U.S. dollar index (1973=100);
- $a_w$ Sectoral labor coefficient;
- *Kw* Multiple that relates average income to wage per hour;
- μ M1 multiplier;

$\alpha$ Share of monetary base on the sum base + debt;

$V$ M1 income velocity.

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